Applications: Bargaining Model of War

Why are there wars?

The next few weeks of class will propose many explanations.

This is not a settled question, but theories can still be useful.

- Today: bargaining failures because of indivisible resources, uncertainty, shifting power.
- Later: Domestic political incentives.
- Later: Differences between democracies and autocracies
- Later: Leader psychology

Why are there wars?

A simple bargaining model shows why war is more surprising than our intuition suggests.

This approach takes (possibly) Realist assumptions and applies game theory. We'll identify three "rationalist" causes of war:

- Indivisible Goods
- Uncertainty about costs of war
- Shifting power

Fearon, James D. "Rationalist explanations for war." *International Organization* 49.3 (1995): 379-414.

Basics of the model

Assume that states are engaged in zero-sum bargaining to divide territory.

Simplification:

- Actors: two states
- Interests: to get maximum territory
- Interaction: sequence of proposals followed by a lottery
- Institutions: rules about bargaining

This is not fully realistic, but that's not the point.

Basics of the model

Utility/Payoff: Amount of satisfaction received from a specified outcome

Expected Utility: Average amount of utility from each possible outcome weighted by that outcome's probability of occurring

Discount Factor: Present value of utility received in the next period (i.e., today's value of future payoffs)

Basics of the model

The model treats war as a lottery.

Two (mathematically identical) interpretations of war outcomes

1. the probability a state gains the entire resource
2. the proportion of the resource the state gets

Equivalent in terms of **expected utility**

Basics of the model

War isn't free:

1. It costs money to fight. Troops die, airplanes destroyed, people get injured.
2. Also destroys the resource that states are battling over!

Basics of the model

Two states, A and B.

Bargaining over a resource worth one "unit" (this just makes the math easier).

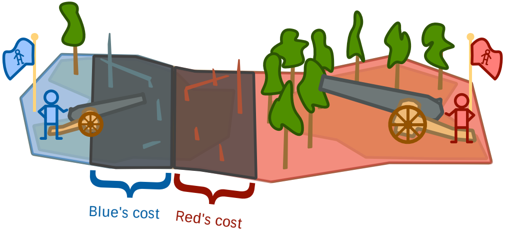
- Power. Expected amount of resource an actor gets following war: p.
- Costs of war: $c_A, c_B > 0$

Bargaining protocol:

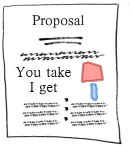
1. A makes a demand to B, x, giving B $1 - x$.
2. B decides to accept or reject the demand.

Basics of the model

Resource (land), Power (cannons), and Costs (destruction)

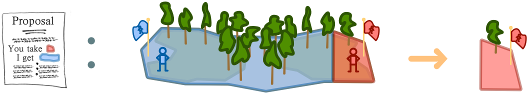


Demand x is a a proposal from Blue on how to divide

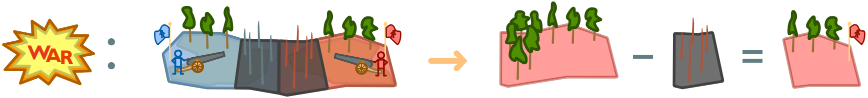


Basics of the model

Consider a demand x. Expected utility to B of accepting: $1 - x$



Expected utility to B of rejecting: $(1 - p) \times 1 - c_B$



Should B accept?

- Accept if $1 - x \geq 1 - p - c_B$
- Accept if $p + c_B \geq x$

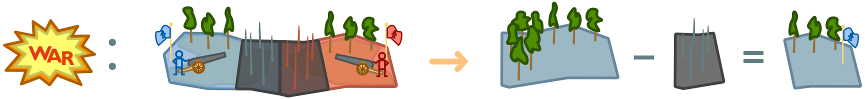
B's "decision-rule": **Reject** if $p + c_B < x$

Basics of the model

What should A demand in order to maximize its utility?

Consider A's expected utility of war.

- $p - c_A$



Thus A prefers having an accepted demand over war that the demand gives them more than the expected utility of war:

$$x \geq p - c_A$$

Basics of the model

A prefers having a demand x accepted to war if $x \geq p - c_A$.

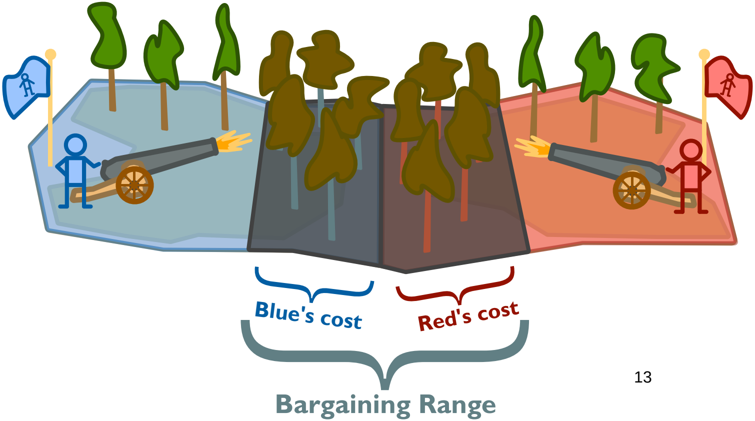
B prefers war to accepting a demand x to war if $p + c_B < x$

Key question: Are there values of x such that both prefer peace to war?
Answer: **YES** if the costs of war are positive.

- $x \in p - c_A, p + c_B$

This "range" is called the **bargaining range**.

Basics of the model



Basics of the model

But this analysis suggests that bargaining should just follow the distribution of power, and this should be peaceful.

But we know this is not right, because wars happen.

Indivisible Goods

Indivisible Goods

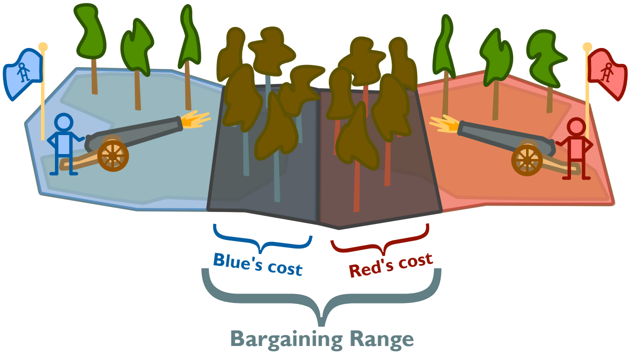
The bargaining range is finite.

In our model, it is $x \in p - c_A, p + c_B$

If the thing states are fighting over is:
1) worth more than the bargaining range
2) indivisible

. . . then war results.

Indivisible Goods

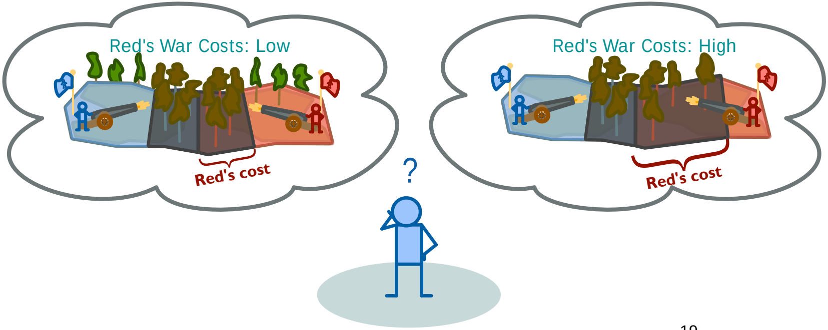


What are some examples in IR?

Uncertainty about costs of war

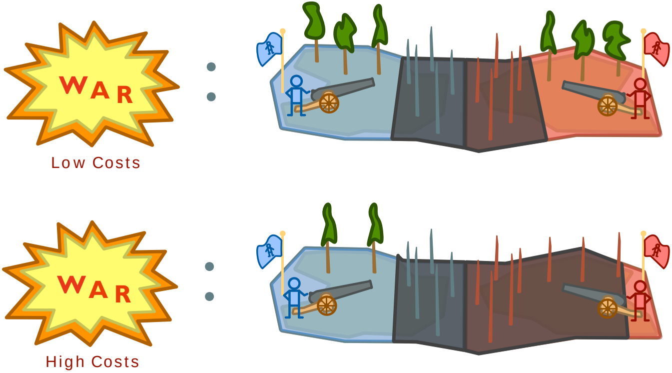
Uncertainty about costs of war

What if the costs of war (c_A, c_B) of the other state not known.



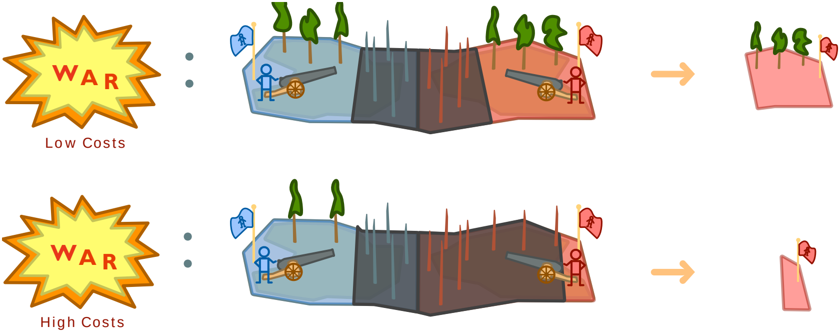
Uncertainty about costs of war

Size of costs affects the size of the bargaining range



Uncertainty about costs of war

Bargaining range size affects payoffs



Uncertainty about costs of war

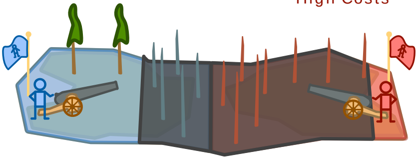
Because of this payoff variation, states have incentives to lie (under-report) their costs of war

$\rightarrow$ incentive to misrepresent size of your costs

Uncertainty about costs of war

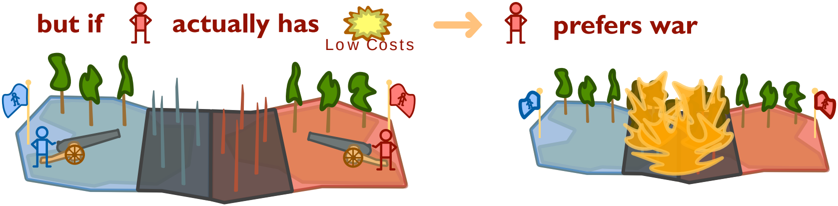
Incomplete information plus incentives to misrepresent can foster war

if thinks has High Costs → **makes ungenerous proposal**



Uncertainty about costs of war

Incomplete information plus incentives to misrepresent can foster war



Is credible communication possible?

Incomplete information can generate war because states cannot credibly communicate their costs of war.

- **Cheap talk:** No costs to bluffing about my costs

Is credible communication possible?

Costly Signaling: Communication can succeed if talk isn't cheap. If signals carry costs, then they can credibly reveal information.

Examples

- **Sinking costs:** Moving an army into position to attack takes resources and is costly. This can reveal a state's resolve.
- **Tying hands:** Leader promises citizens that she will not back down to a challenger. Backing down will now carry costs for her (re-election).

Is credible communication possible?

Sufficiently costly signals can separate different types of states.

Only highly resolved states will send these very costly signals.

Less costly signals cannot separate high resolve states from low resolve states.

Shifting Power

Shifting Power

Logic:

- Rising states cannot *credibly commit* to uphold bargains
- Declining states fear future unfavorable bargains
- For a severe enough power shift, a declining state does best by fighting when still at its strongest

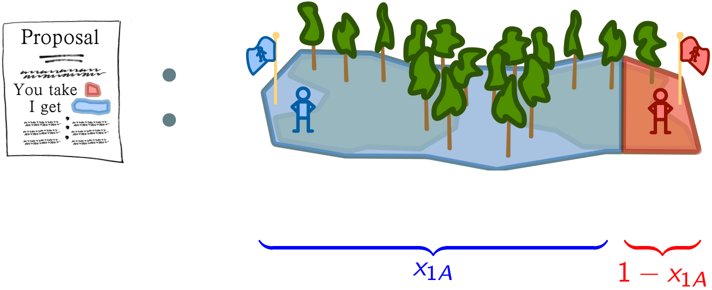
Multi-round bargaining

Lets use the same basic bargaining model as before but add:

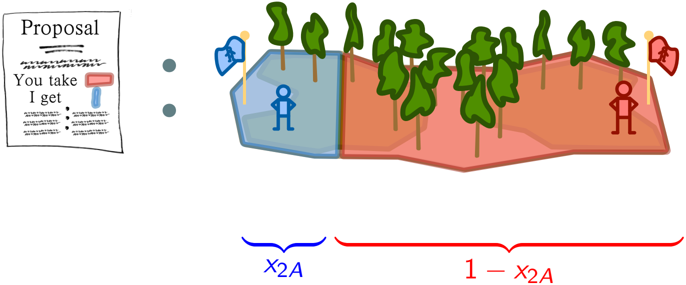
- Multiple rounds of bargaining (x_{1A}, x_{2A})
- Shifting power (p_1, p_2)

Bargaining occurs in two rounds.

In the first period player A makes a demand, x_{1A}, to player B. This leaves player B with $1 - x_{1A}$.



If the first period demand is accepted, then in the second period there is another bargaining stage, where player A makes a demand x_{2A}.



War in period 1 or 2?

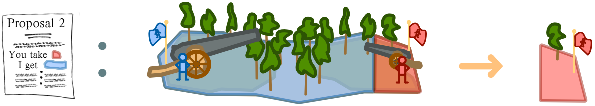
If the first period demand is **rejected** then **war** occurs.

Player A gets p_1 and player B gets $1 - p_1$.
But both players pay costs $c_A = c_B$.

If the offer is rejected in the second period, both players pay the cost and A gets p_2 and B gets $1 - p_2$.

Backwards Induction

Assume that states “look ahead” into the future. If Player B expects a large unfavorable power shift (and thus small payoff) in period 2:



Then B starts war even with the most generous possible first round offer:

Player A can not offer (demand little) enough to make player B accept the demand in light of what they expect to get in period 2.

Shifting Power

Reviewing the logic:

- Rising states cannot *credibly commit* to uphold bargains
- Declining states fear future unfavorable bargains
- For a severe enough power shift, a declining state does best by fighting when still at its strongest

Lots of things like this. "Time inconsistency problems" in

- saving money
- working out
- party platform vs. policy
- dating and marriage?

Bargaining and War

War is inefficient, yet we observe war in the world. Why?

- Indivisible Goods
- Uncertainty about costs of war
- Shifting power

Shifting Power: more detailed derivation

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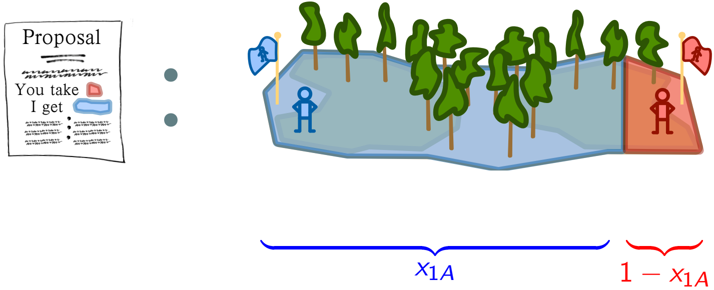

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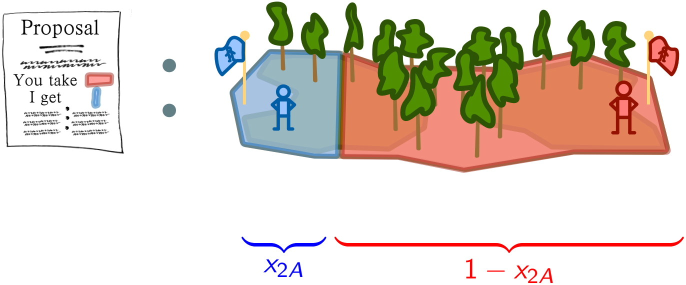

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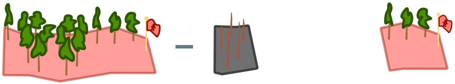
If the offer is rejected in the second period, both players pay the cost and A gets p_2 and B gets $1 - p_2$.

Backwards Induction

Assume that states "look ahead" into the future, and condition their current decisions on what they think will happen in the future.

Last decision by actor B. In Period 2 player B accepts demand x_{2A} (rather than start war)

$$\underbrace{1 - p_2 - c_B}_{\text{Expected Payoff from Fighting}} \leq \underbrace{1 - x_{2A}}_{\text{Payoff from Proposal}} .$$



Thus if $x_{2A} \leq p_2 + c_B$ then actor B accepts the demand.

In period 2 player A's expected utility from war is $p_2 - c_A$.
Hence A will prefer to make a demand $x_{2A} = p_2 + c_B$ if

$$\underbrace{p_2 + c_B}_{\text{Payoff from Proposal}} > \underbrace{p_2 - c_A}_{\text{Expected Payoff from Fighting}} .$$

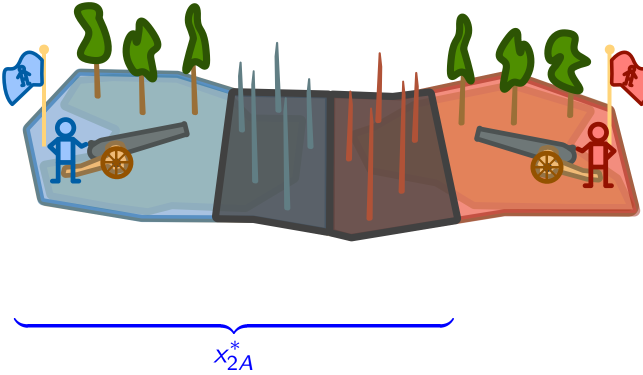
This holds because we assume $c_B, c_A \geq 0$.

They can't make more than this demand because then it will be rejected.

So, the **optimal demand in the second period is**

$$x^*_{2A} = p_2 + c_B.$$

Optimal demand in the second period is $x^*_{2A} = p_2 + c_B$. This leaves B with $1 - x^*_{2A}$, equal to its expected payoff for war.



Now consider period 1. B's utility from rejecting the proposal in period 1 is

$$\underbrace{(1-p_1)-c_B}_{\text{Round 1}}+\delta\underbrace{(1-p_1)}_{\text{Round 2}},$$

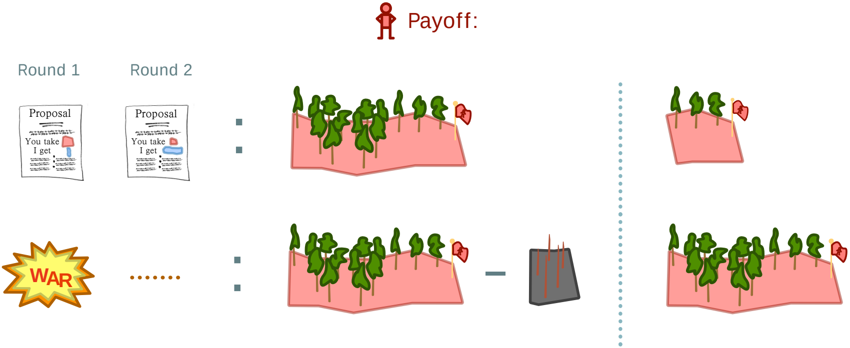
where δ represents the discount rate for the second period, or the likelihood that the second period is played.

We will assume that $\delta = 1$ for simplicity

Player B's utility from accepting a demand x_{1A} is equal to

$$\underbrace{1 - x_{1A}}_{\text{Round 1}} + \underbrace{\delta(1 - p_2 - c_B)}_{\text{Round 2}}.$$

Round 1 Round 2


Payoff:
Round 1
Round 2
Proposal
You take
I get
Proposal
You take
I get
WAR

Thus player B will reject the demand x_{1A} if

$$\begin{aligned} \underbrace{(1-p_1) - c_B + \delta(1-p_1)}_{\text{Expected Payoff from Fighting}} &> \underbrace{1 - x_{1A} + \delta(1-p_2-c_B)}_{\text{Expected Payoff from Bargaining}} \\ x_{1A} &> p_1 + \delta p_1 - \delta p_2 + c_B - \delta c_B \end{aligned}$$

Hence they will be indifferent if $x_{1A} = p_1 + \delta p_1 - \delta p_2 + c_B - \delta c_B$.

$x_{1A} = 2p_1 - p_2$ for $\delta = 1$

Now consider A's expected utility in period 1. If they have the proposal rejected their expected utility is

$$\underbrace{p_1 - c_A}_{\text{Round 1}} + \underbrace{\delta p_1}_{\text{Round 2}}.$$

If they have some demand x_{1A} accepted then they get

$$\underbrace{x_{1A}}_{\text{Round 1}} + \underbrace{\delta(p_2 + c_B)}_{\text{Round 2}}.$$

A will want their first period demand accepted if

$$\underbrace{x_{1A} + \delta(p_2 + c_B)}_{\text{Expected Payoff from Accepting}} \geq \underbrace{p_1 - c_A + \delta p_1}_{\text{Expected Payoff from War}}$$

$$x_{1A} \geq p_1 + \delta p_1 - \delta p_2 - c_A - \delta c_B$$

$$x_{1A} \geq 2p_1 - p_2 - c_A - c_B; \text{for } \delta = 1$$

Now note that the right hand side of this is almost identical to what will make B indifferent, except that it is slightly smaller.

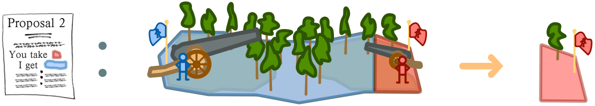
Hence A will make demand:

- $x_{1A}^* = p_1 + \delta p_1 - \delta p_2 + c_B - \delta c_B$
- $x_{1A}^* = 2p_1 - p_2$; for $\delta = 1$
- $x_{2A}^* = p_2 + c_B$

First period demands will always be rejected (preventive war) when x^*_{1A} is less than 0. (when $p_2 > 2p_1$)

$\rightarrow$ When player A can not offer (demand little) enough to make player B accept the demand in light of what they expect to get in period 2.

If Player B expects a large unfavorable power shift (and thus small payoff) in period 2:



Then B starts war even with the most generous possible first round offer:

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