Class 10 in-class problems, 18.05, Spring 2022

Concept questions

Concept question 1. Is it a statistic?

You believe that the lifetimes of a certain type of lightbulb follow an exponential distribution with parameter λ. To test this hypothesis you measure the lifetime of 5 bulbs and get data $x_1, \ldots x_5$.

Which of the following are statistics?

(a) The sample average $\overline{x} = \frac{x_1+x_2+x_3+x_4+x_5}{5}$.

(b) The expected value of a sample, namely $1/\lambda$.

(c) The difference between $\overline{x}$ and $1/\lambda$.

Board questions

Problem 1. Coins

(a) A box contains 3 coins. They land heads with, respectively, probability $p = 1/3,\ 1/2,\ 2/3$.

A coin is taken from the box. The mystery coin is tossed 80 times, resulting in 49 heads and 31 tails.

What is the likelihood of this data for each type of coin? Which coin gives the maximum likelihood?

(b) Now suppose you found a bent coin. It has an unknown probability p of landing heads. To estimate p you toss it 80 times getting 49 heads. Find the likelihood and log likelihood functions given this data. What is the maximum likelihood estimate for p?

Problem 2. Continuous likelihood

For continuous likelihood: use the pdf instead of the pmf

Box of light bulbs.

Lifetime of each bulb $\sim \exp(\lambda)$, with unknown parameter λ.

For multiple independent data points, the likelihood is the product of the individual likelihoods.

(a) We test 5 light bulbs and find they have lifetimes of 2, 3, 1, 3, 4 years respectively. We assume the tests are independent.

(i) Find the likelihood and log likelihood functions (as functions of λ.)

(ii) What is the maximum likelihood estimate (MLE) for λ?

Reminder: An exponential distribution has pdf $f(x|\lambda) = \lambda e^{-\lambda x}$

(b) Suppose we test 5 bulbs and find they have lifetimes x_1, x_2, x_3, x_4, x_5 years respectively. Redo Part (a) using these lifetimes.

Extra

Cilantro problem In the Cilantro experiment, assume 55 out of 100 people said Cilantro tastes like soap. Find the maximum likelihood estimate for p, the true proportion of people who feel that way.

18.05 Introduction to Probability and Statistics
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