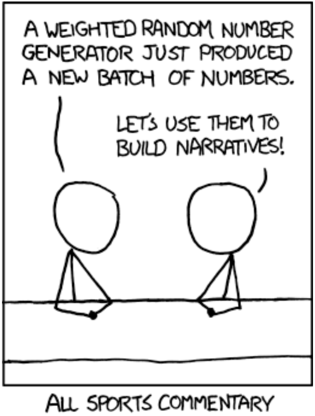
Welcome to 18.05
Introduction to Probability and Statistics
Spring 2022



https://xkcd.com/904/

Agenda

- Introductions and class description
- Administrative stuff
- Begin probability: sets and counting

Websites

You should have received this information in an email.
If not let us know.

- MITx:
  - If you registered for 18.05 you should be able to see the class.
  - Site will have all reading materials, problem sets, etc.
  - Lecture slides and problems will be posted before class. Solutions right after class.
- Canvas: Has links to everything.
- Gradescope: For turning in psets and quizzes
- Piazza: Sign up – there is a link on our Canvas site.
  For asynchronous help: everyone can ask and answer questions. The teaching staff will also monitor the site and respond to questions.

Active Learning

Read the 'Calendar and Information' section on our MITx site.

All the educational research shows two things

- Better and more long lasting educational gains.
- Many students doubt this at first.

Before class

- Reading and reading questions.
- Reading questions count toward grade.
- Lecture will assume you've done the reading.

In class:

- Combination of lecture and problem solving
- We *won't* assume you've completely mastered the reading.
  - Will assume a level of familiarity.
  - Use the Piazza discussion board – links on MITx and Canvas.
  - Bring questions to class.

Class

Read the 'Calendar and Information' section on our MITx/18.05r site.

Class Time

- TR: Lecture/concept (clicker) questions/board questions
  - Participation on clicker questions counts towards your grade
  - Will use MITx for clicker questions –requires a computer or phone.
  - No computer or phone use –except for clicker questions– in class on TR.
- F: Studio – bring your laptop

In-class Groups

- Groups of 3.
- You will be able to choose your own group.
- If you need to find a group or your group needs a third person let us know and we'll help.

R: for computation, simulation and visualization

- will teach you everything you need
- no hardcore programming.

Problem Sets

- Usually due on Mondays
- Turn in to Gradescope by 10 PM
- You'll be able to check your numerical answers to problems on our MITx site before the due date.
- Problem sets will be graded on the logic and explanation of your answer.

R, Piazza

R

- Free open source package.
- Very easy to use and install.
- Instructions and a link for this are on MITx/18.05r.

Piazza

- We will use the Piazza discussion forum.
- Mostly for students to ask questions of each other.
- Sign up by following the link from our MITx site.

Calendar, Information, Policies and Goals

Everything we just went over and more is in the

Calendar and Information section of MITx/18.05r

For Next Time

- Familiarize yourself with the MITx/18.05r site
- Install R and R Studio

- Sign up for Piazza and join our class. (Link on our Canvas site)
- Read class 1 notes (a summary of what we'll do today)
- Go through the class 2 sequence and answer the reading questions

Platonic Dice

4, 6, 8, 12, 20-sided

Probability vs. Statistics

Different subjects: both about random processes

Probability

- Logically self-contained
- A few rules for computing probabilities
- One correct answer

Statistics

- Messier and more of an art
- Seek to make probability based inferences from experimental data
- No single correct answer

Counting: Motivating Examples

What is the probability of getting exactly 1 heads in 3 tosses of a fair coin?

Poker Hands

Deck of 52 cards

- 13 *ranks*: 2, 3, ... , 9, 10, J, Q, K, A
- 4 *suits*: $\heartsuit, \spadesuit, \diamondsuit, \clubsuit,$

Poker hands

- Consists of 5 cards
- A *one-pair* hand consists of two cards having one rank and the remaining three cards having three other ranks
- Example: $\{2\heartsuit, 2\spadesuit, 5\heartsuit, 8\clubsuit, \mathsf{K}\diamondsuit\}$

Concept Question 1

A *one-pair* hand consists of two cards having one rank and the remaining three cards having three other ranks

The probability of a one-pair hand is:

(1) less than 5%
(2) between 5% and 10%
(3) between 10% and 20%
(4) between 20% and 40%
(5) greater than 40%

Sets in Words

Old New England rule: don't eat clams (or any shellfish) in months without an 'r' in their name.

- $S =$ all months
- $L =$ the month has 31 days
- $R =$ the month has an 'r' in its name

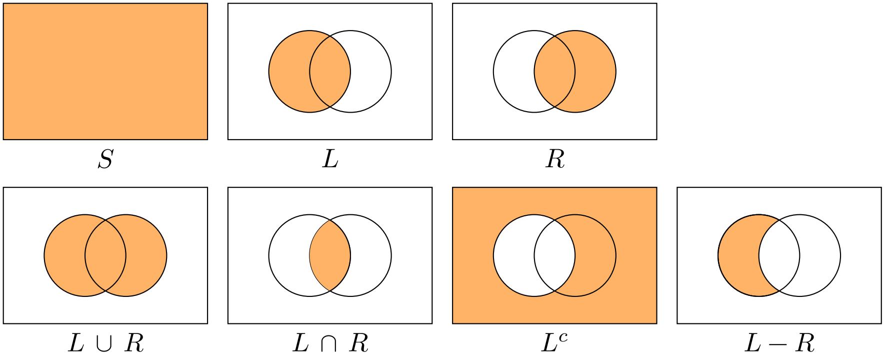
$S = \{$Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec$\}$

$L = \{$Jan, Mar, May, Jul, Aug, Oct, Dec$\}$

$R = \{$Jan, Feb, Mar, Apr, Sep, Oct, Nov, Dec$\}$

$L \cap R = \{$Jan, Mar, Oct, Dec$\} =$ months with 31 days and an 'r'

Visualize Set Operations with Venn Diagrams



Product of Sets

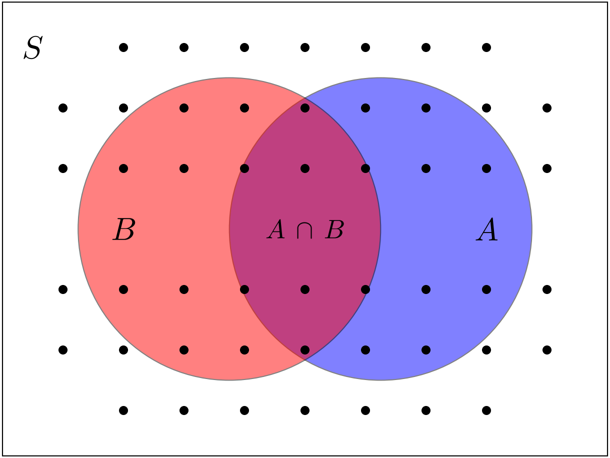
$$S \times T = \{\text{pairs } (s, t) \text{ with } s \text{ in } S,\ t \text{ in } T\}$$

More simply: $S \times T = \{(s, t)\}$

SIZE of $S \times T$ = (size of S) $\cdot$ (size of T)

$$|S \times T| = |S| \cdot |T|.$$

Inclusion-Exclusion Principle



$$|A \cup B| = |A| + |B| - |A \cap B|$$

Board Question 1

A band consists of singers and guitar players:

7 people sing, 4 play guitar, 2 do both

How many people are in the band?

Rule of Product

Example

$$3 \text{ shirts, } 4 \text{ pants} = 12 \text{ outfits}$$

$$(\text{set of shirts}) \times (\text{set of pants}) = \text{set of outfits}$$

$$|S| \cdot |T| = |S \times T|$$

(More powerful than it seems.)

Concept Questions: DNA

1. DNA is made of sequences of nucleotides: A, C, G, T.
How many DNA sequences of length 3 are there?

(i) 12 (ii) 24 (iii) 64 (iv) 81

Concept Questions: DNA

1. DNA is made of sequences of nucleotides: A, C, G, T. How many DNA sequences of length 3 are there?

(i) 12 (ii) 24 (iii) 64 (iv) 81

2. How many DNA sequences of length 3 are there with no repeats?

(i) 12 (ii) 24 (iii) 64 (iv) 81

Board Question 2

There are 5 Competitors in an Olympics 100m final.

How many ways can gold, silver, and bronze be awarded?

Board Question 3

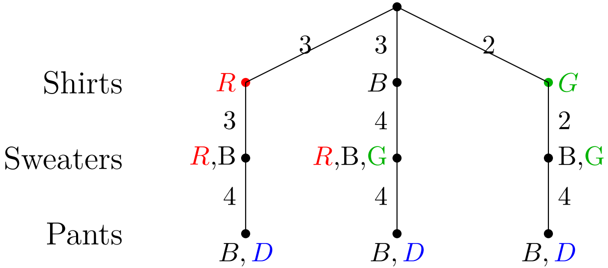
I won't wear green and red together; I think black or denim goes with anything; Here is my wardrobe.

Shirts: 3B, 3R, 2G; sweaters 1B, 2R, 1G; pants 2D,2B.

How many different outfits can I wear?

Solution

Solution: Suppose we choose shirts first. Depending on whether we choose red compatible or green compatible shirts there are different numbers of sweaters we can choose next. So we split the problem up before using the rule of product. A multiplication tree is an easy way to present the answer.



Multiplying down the paths of the tree:
Number of outfits $= (3 \times 3 \times 4) + (3 \times 4 \times 4) + (2 \times 2 \times 4) = 100$

Permutations

Lining things up. How many ways can you do it?

'abc' , 'cab' are 2 of the 6 permutations of $\{a, b, c\}$

'ad' , 'da' , 'bc' are three of the twelve permutations of two things from $\{a,b,c,d\}$

Permutations of k from a set of n

Give all permutations of 3 things out of $\{a, b, c, d\}$

Permutations of k from a set of n

Give all permutations of 3 things out of $\{a, b, c, d\}$

$$\begin{array}{cccccc} abc & abd & acb & acd & adb & adc \\ bac & bad & bca & bcd & bda & bdc \\ cab & cad & cba & cbd & cda & cdb \\ dab & dac & dba & dbc & dca & dcb \end{array}$$

Would you want to do this for 7 from a set of 10?

Combinations

Choosing subsets – order doesn't matter.

How many ways can you do it?

Combinations of k from a set of n

Give all combinations of 3 things out of $\{a, b, c, d\}$

Answer: {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}

Permutations and Combinations

$${}_nP_k = \text{number of permutations (ordered lists)}$$
$$\text{of } k \text{ things from } n$$

$${}_nC_k = \binom{n}{k} = \text{number of combinations (subsets)}$$
$$\text{of } k \text{ things from } n$$

$${}_nP_k = \frac{n!}{(n-k)!} \qquad \binom{n}{k} = {}_nC_k = \frac{{}_nP_k}{k!} = \frac{n!}{(n-k)!\,k!}$$

Proof: Rule of product!

Permutations and Combinations

$$\begin{array}{cccccc} abc & acb & bac & bca & cab & cba \\ abd & adb & bad & bda & dab & dba \\ acd & adc & cad & cda & dac & dca \\ bcd & bdc & cbd & cdb & dbc & dcb \end{array} \qquad \begin{array}{c} \{a,b,c\} \\ \{a,b,d\} \\ \{a,c,d\} \\ \{b,c,d\} \end{array}$$

Permutations:

$_4P_3$

Combinations:

$\binom{4}{3} = {}_4C_3$

$$\binom{4}{3} = {}_4C_3 = \frac{_4P_3}{3!} = \frac{4!}{3!\,1!}$$

Board Question 4

(a) Count the number of ways to get exactly 3 heads in 10 flips of a coin.

(b) For a fair coin, what is the probability of exactly 3 heads in 10 flips?