

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
15.053 – Optimization Methods in Management Science (Spring 2007)

Problem Set 8

Due April 19th, 2007 at 4:30 pm.
You will need 120 points out of 140 to receive a grade of 5.

All Problems are Worth 20 Points

Problem 1: Caterer problem.

As part of its food service, a caterer needs d_j napkins for each day of the upcoming week. He can buy new napkins at the price of α cents each or have his soiled napkins laundered. Two types of laundry service are available: regular and expedited. The regular laundry service requires two working days and costs β cents per napkin, and the expedited service requires one working day and costs γ cents per napkin ($\gamma > \beta$). The problem is to determine a purchasing and laundry policy that meets the demand at the minimum possible cost. Formulate this problem as a minimum costs flow problem. (*Hint:* Define a network on 15 nodes, 7 nodes corresponding to soiled napkins, 7 nodes corresponding to fresh napkins, and 1 node for the supply of fresh napkins.)

Problem 2: Passenger routing

United Airlines has six daily flights from Chicago to Washington. From 10 A.M. until 8 P.M., the flights depart every 2 hours. The first three flights have a capacity of 100 passengers and the last three flights can accommodate 150 passengers each. If overbooking results in insufficient room for a passenger on a scheduled flight, United can divert a passenger to a later flight. It compensates any passenger delayed by more than 2 hours from his or her regularly scheduled departure by paying \$200 plus \$20 for every hour of delay. United can always accommodate passengers delayed beyond the 8 P.M. flight on the 11 P.M. flight of another airline that always has a great deal of spare capacity. Suppose that at the start of a particular day the six United flights have 110, 160, 103, 149, 175, and 140 confirmed reservations.

Part A:

Formulate the problem of determining the most economical passenger routing strategy as a minimum cost flow problem.

Part B:

Formulate it as a linear program

Problem 3:

Consider the Minimum Cost Network Flow problem shown below. The numbers next to each arc represent arc costs. Each capacity is infinite. Consider the tree indicated by thicker lines, which are the arcs of the initial basis.

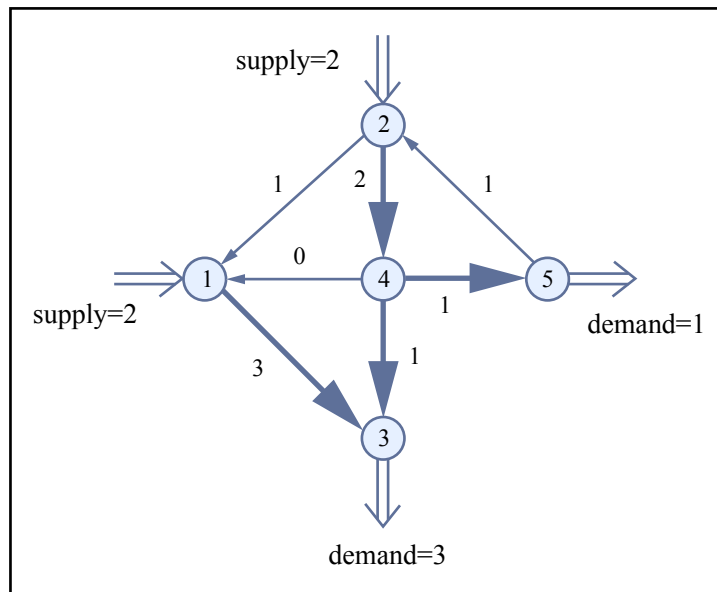


Image by MIT OCW.

Part A:

What are the values of the arc flows corresponding to this basic solution?

Part B:

Calculate the simplex multipliers for each node (Note: Please double check your simplex multipliers because they are critical to your calculations in part c)).

Part C:

Find the value of the reduced cost of each nonbasic arc, and explain why we have an optimal solution.

Part D:

By how much can we decrease c_{52} and still maintain optimality?

Problem 4: Spanning Trees

Lisa Raymond needs your help!

She has the following graph:

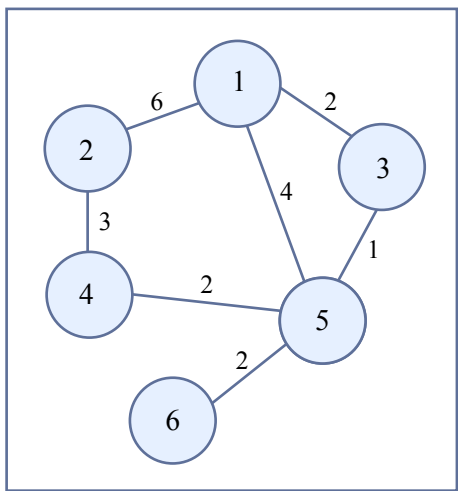


Image by MIT OCW.

Part A:

How many arcs will be in each spanning tree for the above graph? (Will it be the same number for each spanning tree?)

Part B:

Identify a spanning tree.

Part C:

How many different spanning trees does the above graph have?

Problem 5: Network Simplex

Consider the following network:

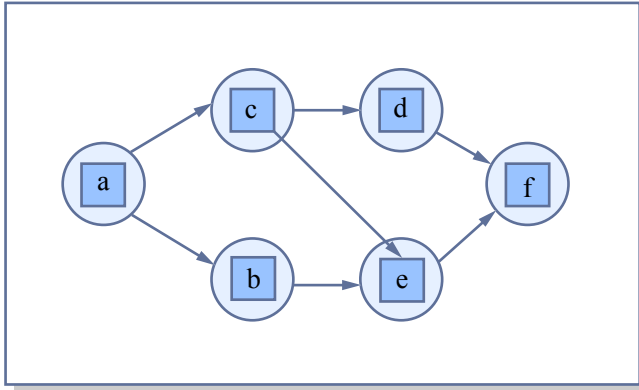
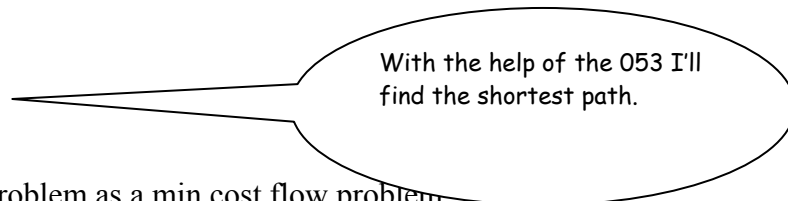


Image by MIT OCW.

The following are the arc lengths:

- (a,b)= 3
- (a,c)= 4
- (c,e)= 2
- (b,e)= 3
- (c,d)= 4
- (d,f)= 2
- (e,f)= 2

Turkey Tim needs to travel from node a to node f in the shortest path in order to get to the other side. (And you thought that was only for chickens.)



Part A:

Formulate this problem as a min cost flow problem.

Part B:

Solve using the network simplex algorithm. Use arcs

- (a,c)
- (c,d)
- (d,f)
- (a,b)

(c,e)

as your initial spanning tree basis.

For each iteration:

- Calculate the spanning tree flows.
- Calculate the simplex multipliers.
- Calculate the reduced costs of all nonbasic arcs.
- For the entering arc determine the maximum amount of flow that can be sent in the entering arc and thus around the basic cycle.
- Determine the exiting arc

Problem 6

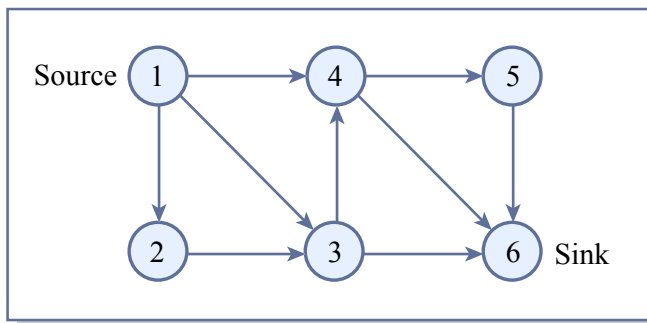


Image by MIT OCW.

Consider the network given in the figure; assume that each arc has capacity 1.

Part A:

Compute the maximum number of arc-disjoint paths from the source node to the sink node. (You might do so by inspection.)

Part B:

Enumerate all $s-t$ cuts in the network. For each $s-t$ cut $[S, S]$, list the node partition and the sets of forward and backward arcs.

Part C:

Verify that the maximum number of arc-disjoint paths from node s to node t equals the minimum number of forward arcs in an $s-t$ cut.

Problem 7

Consider the matrix rounding problem given below (Read Note A). We want to round each element in the matrix, and also the row and column sums, to the nearest multiple of 2 so that the sum of the rounded elements in each row equals the rounded row sum and the sum of the rounded elements in each column equals the rounded column sum.

			Row sum	
	7.5	6.3	15.4	29.2
	3.9	9.1	3.6	16.6
	15.0	5.5	21.5	42.0
Column sum	26.4	20.9	40.5	

Image by MIT OCW.

Formulate this problem as a maximum flow problem and solve it.

Challenge Problem I:

Consider a 0-1 matrix H with n_1 rows and n_2 columns. We refer to a row or a column of the matrix H as a line. We say that a set of 1's in the matrix H is *independent* if no two of them appear in the same line. We also say that a set of lines in the matrix is a *cover* of H if they include (i.e., "cover") all the 1's in the matrix. Show that the maximum number of independent 1's equals the minimum number of lines in a cover. (*Hint*: Use the max-flow min-cut theorem on an appropriately defined network.)