Electricity and Magnetism

- Reminder
  - RLC Circuits
  - Resonance
- Today
  - LC circuits / Oscillations
  - Displacement current
  - Maxwell's equations

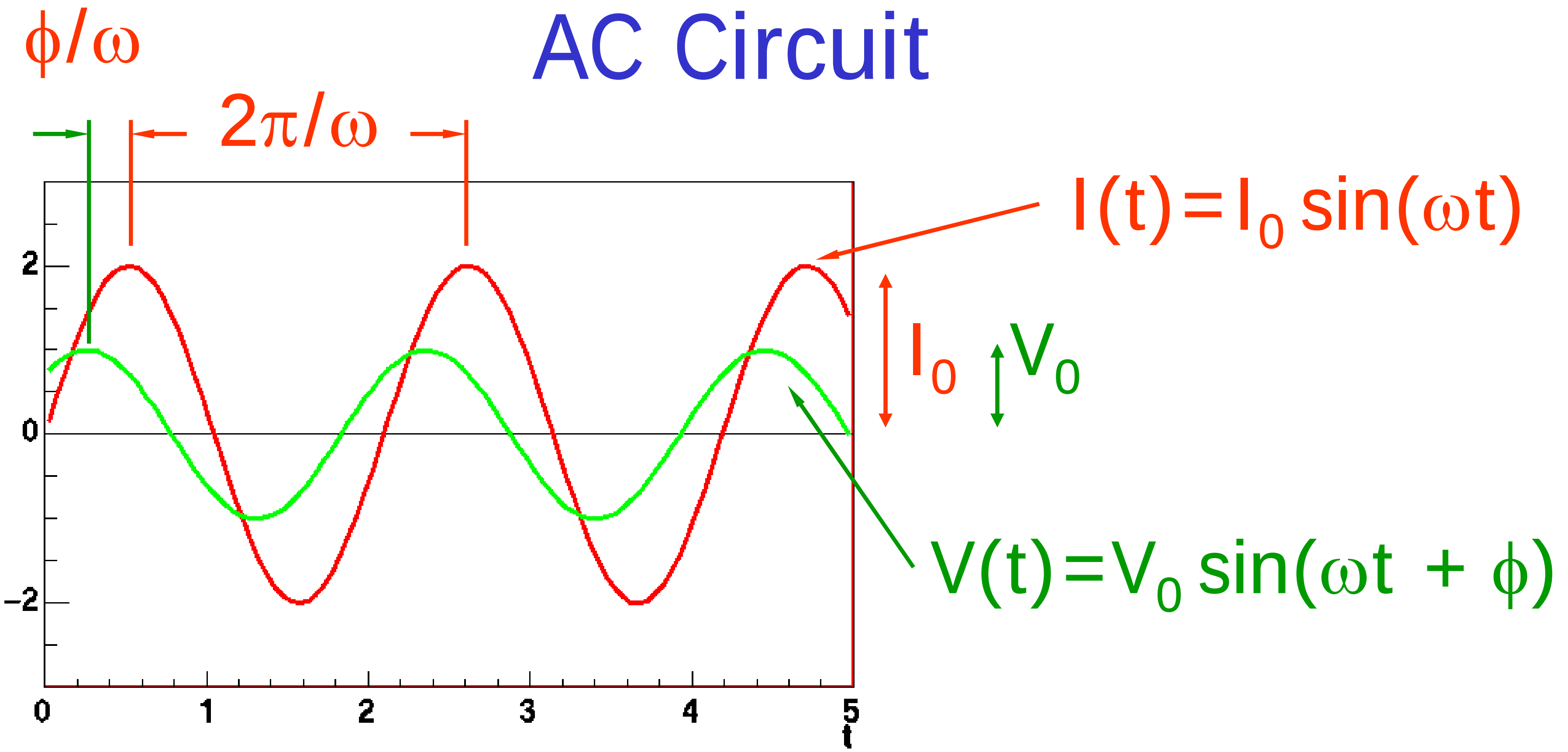
AC Circuit

- AC circuit
  - $I(t) = I_0 \sin(\omega t)$
  - $V(t) = V_0 \sin(\omega t + \phi)$

$\left.\begin{array}{l} I(t) = I_0 \sin(\omega t) \\ V(t) = V_0 \sin(\omega t + \phi) \end{array}\right\}$ same ω!

- Relationship between V and I can be characterized by two quantities
  - Impedance $Z = V_0/I_0$
  - Phase-shift ϕ

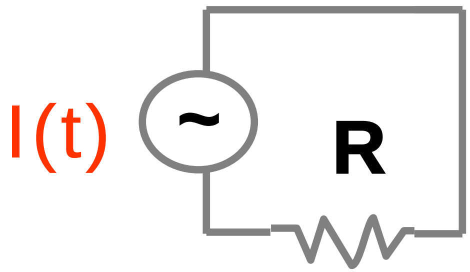
AC Circuit



Impedance $Z = V_0/I_0$

Phase-shift ϕ

First: Look at the components

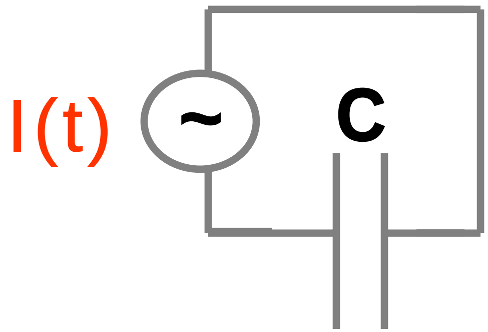


$$V = I\,R$$

$$Z = R$$

$$\phi = 0$$

V and I in phase

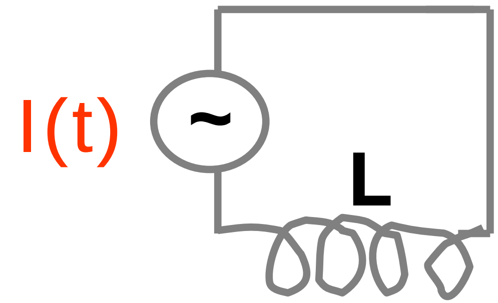


$$V = Q/C = 1/C \int I dt$$

$$Z = 1/(\omega C)$$

$$\phi = -\pi/2$$

V lags I by 90°



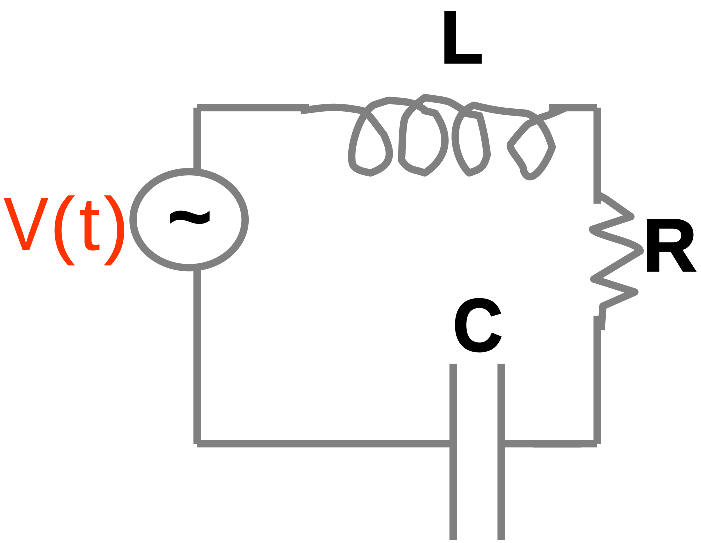
$$V = L\, dI/dt$$

$$Z = \omega L$$

$$\phi = \pi/2$$

I lags V by 90°

RLC Circuit



$$V - L\, dI/dt - IR - Q/C = 0$$

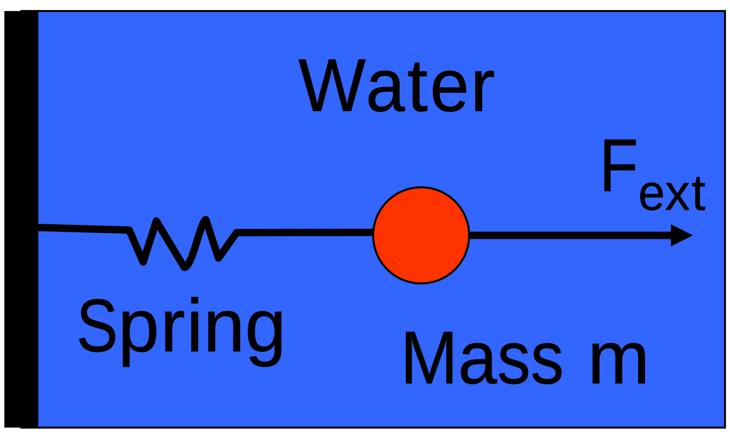
$$L\, d^2Q/dt^2 = -1/C\, Q - R\, dQ/dt + V$$

'Inertia'

'Spring'

'Drag'

$$m\, d^2x/dt^2 = -k\, x - f\, dx/dt + F_{ext}$$



RLC Circuit

$V_0 \sin(\omega t) = I_0\{[\omega L - 1/(\omega C)] \cos(\omega t - \phi) + R \sin(\omega t - \phi)\}$

Solution (requires two tricks):

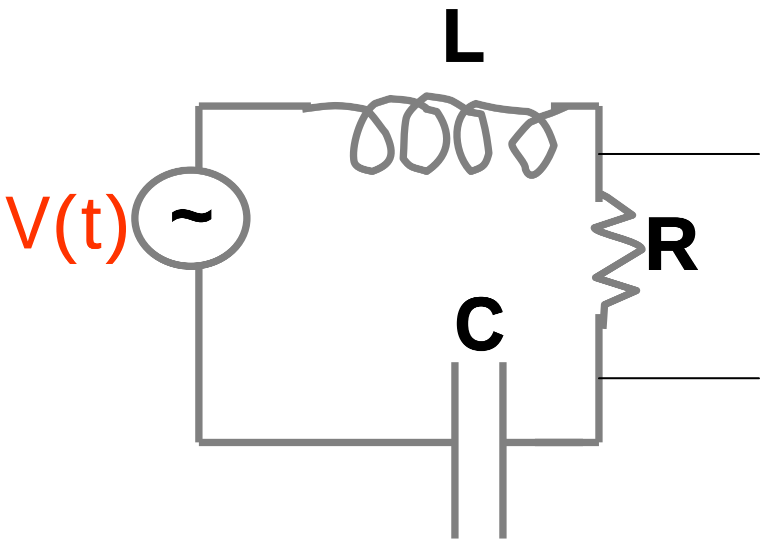
$I_0 = V_0/([\omega L - 1/(\omega C)]^2 + R^2)^{1/2} = V_0/Z$

$\tan(\phi) = [\omega L - 1/(\omega C)]/R$

-> For $\omega L = 1/(\omega C)$, Z is minimal and $\phi = 0$

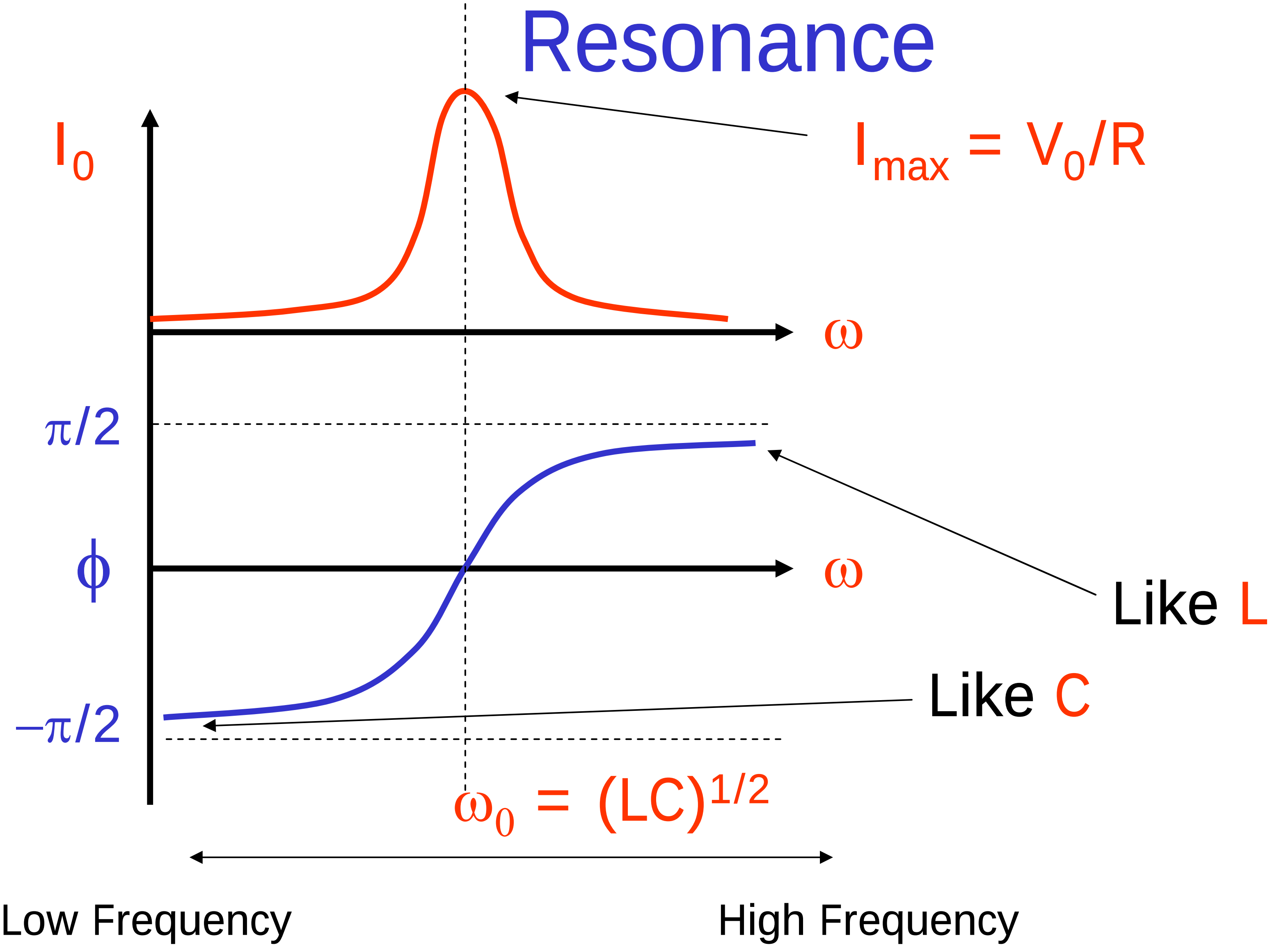
i.e. $\omega_0 = 1/(LC)^{1/2}$ <u>Resonance Frequency</u>

In-Class Demo (on scope)



$V_R(t) \sim I(t)$

Resonance



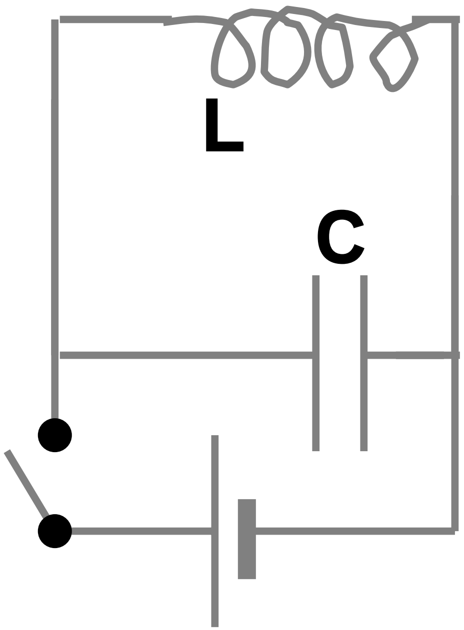
I_0
$I_{max} = V_0/R$
ω
π/2
ϕ
ω
Like L
Like C
−π/2
$\omega_0 = (LC)^{1/2}$
Low Frequency
High Frequency

Resonance

- Practical importance
  - ‘Tuning’ a radio or TV means adjusting the resonance frequency of a circuit to match the frequency of the carrier signal

LC Circuit

- What happens if we open switch?



$$- L\, dI/dt \;-\; Q/C = 0$$

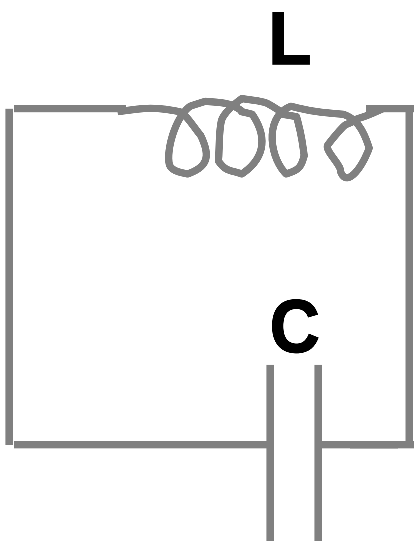
$$L\, d^2Q/dt^2 \;+\; Q/C \;=\; 0$$

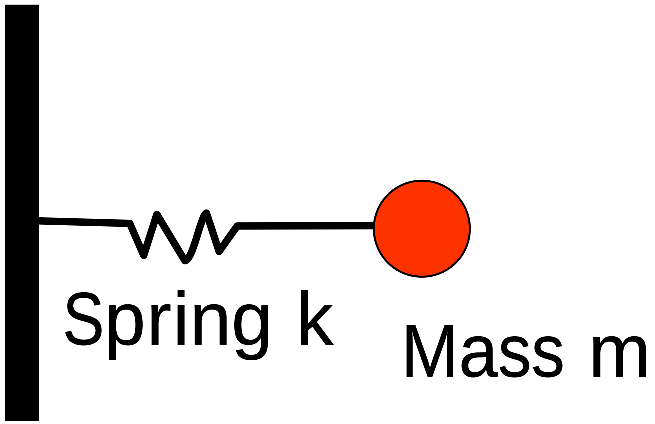
$$\updownarrow$$

$$d^2x/dt^2 \;+\; \omega_0^{\,2}\, x \;=\; 0$$

Harmonic Oscillator!

LC Circuit





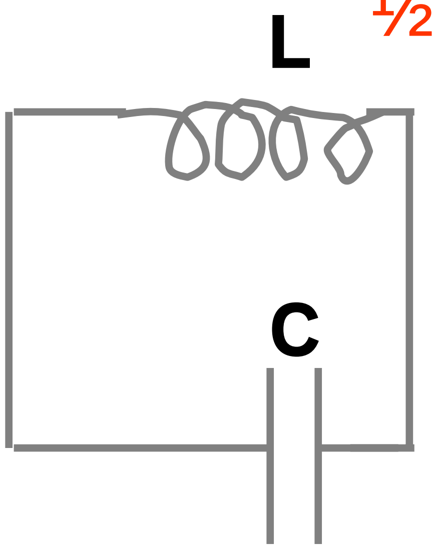
$d^2Q/dt^2 + 1/(LC)\ Q = 0$

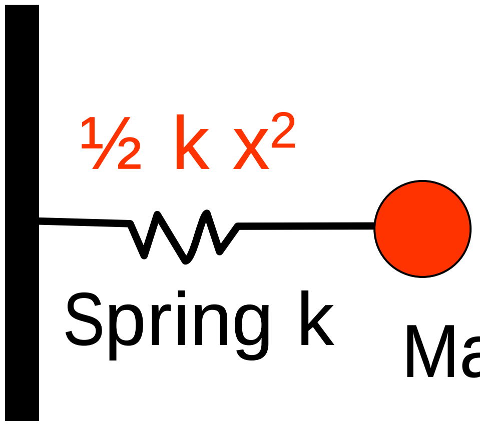
$d^2x/dt^2 + k/m\ x = 0$

$\omega_0^2 = 1/(LC)$

$\omega_0^2 = k/m$

LC Circuit





Energy in E-Field

Oscillation

Energy in B-Field

Potential Energy

Oscillation

Kinetic Energy

Electromagnetic Oscillations

- In an LC circuit, we see oscillations:

Energy in E-Field

$\updownarrow$

Energy in B-Field

- Q: Can we get oscillations without circuit?
- A: Yes!
  - **Electromagnetic Waves**

Maxwell's Equations (almost)

$$\oint_{A_{closed}} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Charges are the source of Electric Flux through close surface

$$\xi = \oint_{L_{closed}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Changing magnetic field creates an electric field

$$\oint_{A_{closed}} \vec{B} \cdot d\vec{A} = 0$$

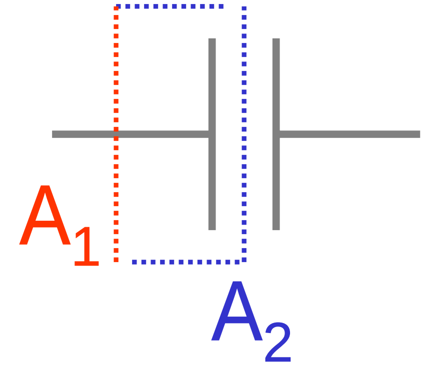
There are no magnetic monopoles

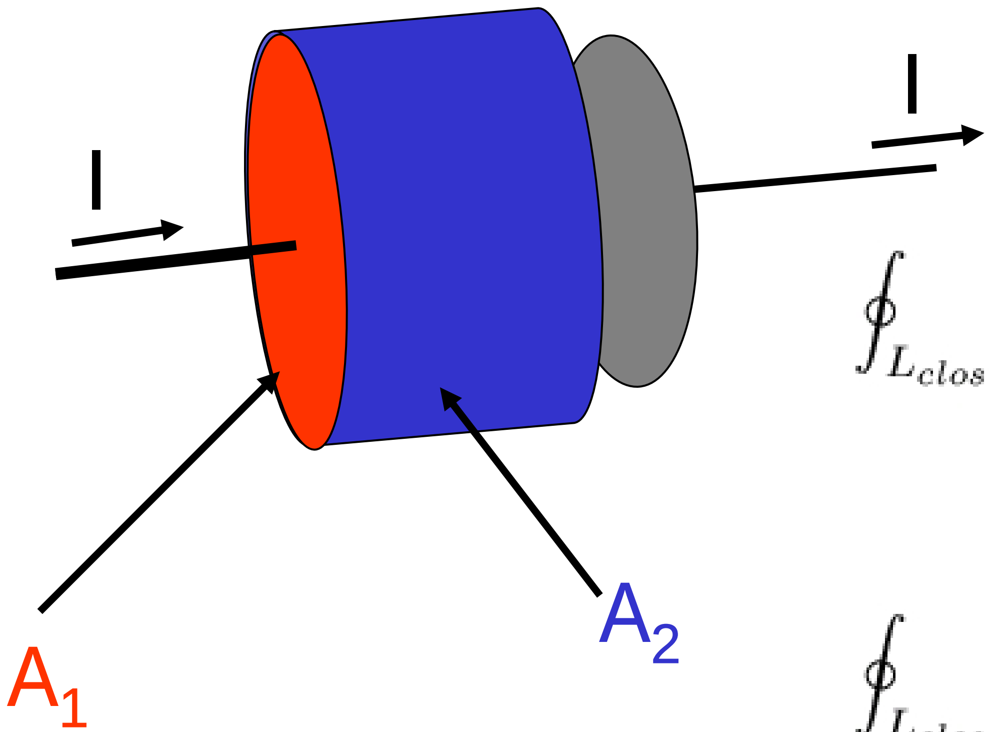
$$\oint_{L_{closed}} \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Moving charges create magnetic field

- Connection between electric and magnetic phenomena
- But not symmetric
- -> James Clerk Maxwell (~1860)

The missing piece





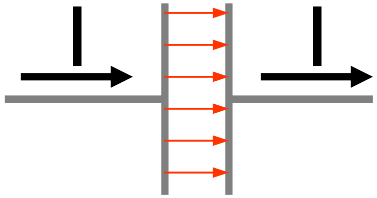
$$\oint_{L_{closed}} \vec{B} \cdot d\vec{l} \;=\; \mu_0 \int_{A1} \vec{J}_1 d\vec{A}$$

=0!

$$\oint_{L_{closed}} \vec{B} \cdot d\vec{l} \;=\; \mu_0 \int_{A2} \vec{J}_2 d\vec{A}$$

Displacement Current

- Ampere's Law broken – How can we fix it?



$Q = C\ V$

Displacement Current $I_D = \varepsilon_0\, d\Phi_E/dt$

Maxwell's Equations

$$
\begin{aligned}
\oint_{A_{closed}} \vec{E} \cdot d\vec{A} &= \frac{Q_{encl}}{\epsilon_0} \\
\oint_{L_{closed}} \vec{E} \cdot d\vec{l} &= -\frac{d\Phi_B}{dt} \\
\oint_{A_{closed}} \vec{B} \cdot d\vec{A} &= 0 \\
\oint_{L_{closed}} \vec{B} \cdot d\vec{l} &= \mu_0 I_{encl} + \mu_0 \, \epsilon_0 \frac{d\Phi_E}{dt}
\end{aligned}
$$

- Symmetry between E and B
  - although there are no magnetic monopoles
- Basis for radio, TV, electric motors, generators, electric power transmission, electric circuits etc

Maxwell's Equations

$$\oint_{A_{closed}} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint_{L_{closed}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint_{A_{closed}} \vec{B} \cdot d\vec{A} = 0$$

$$\oint_{L_{closed}} \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \underbrace{\mu_0 \epsilon_0}_{1/c^2} \frac{d\Phi_E}{dt}$$

- M.E.'s ***predict*** electromagnetic waves, moving with speed of light
- Major triumph of science

Maxwell's Equations

$$\oint_{A_{closed}} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint_{L_{closed}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint_{A_{closed}} \vec{B} \cdot d\vec{A} = 0$$

1/c^2

$$\oint_{L_{closed}} \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\ \epsilon_0 \frac{d\Phi_E}{dt}$$

- Symmetry between E and B
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