

22.01 Problem Set 4 Homework Solutions

1. (2 points) Which can transfer more energy to an electron in a single collision – a proton or an alpha particle? Explain.

Lets calculate the maximum energy transfer in a single collision of a proton or alpha particle with an electron. We can say that the mass of the electron, $m_e = 1$ and the mass of the proton is $m_p = 1836$ and the mass of the alpha is $m_\alpha = 7344$. Solving equation 5.4:

$$Q_p = \frac{4mME}{(M+m)^2} = \frac{4(1)(1836)E}{(1+1836)^2} = 0.0022E$$

$$Q_\alpha = \frac{4mME}{(M+m)^2} = \frac{4(1)(7344)E}{(1+7344)^2} = 5.45 \times 10^{-4} E$$

Therefore the proton will transfer about four times more energy per collision then the alpha particle. This illustrates that an alpha particle, or a proton, or a heavy charged particle will deposit very little of its energy (even at a maximum!) to an electron in a collision. A heavy charged particle will then have many collisions, i.e. ionizations and excitations, before coming to a rest.

2. (2 points) A proton and an alpha particle with the same velocity are incident on a water target. Which will penetrate to a greater depth? Explain.

The range of another particle compared to the range of a proton is given by:

$$R_\alpha(\beta) = \frac{M}{Z^2} R_p(\beta)$$

Notice that β is the component of the equation that is constant, and so this equation holds for a particle at the same velocity of a proton.

For an alpha, $M = 4$, $Z = 2$, and we see that the ranges are equal at the same velocity.

(Remember that even though the velocity is the same, the kinetic energy of the two particles is different.)

3. (3 points) Astatine –211 is an alpha-emitting isotope that is used in radiation therapy by attaching it to a monoclonal antibody. The energy of the ^{211}At alpha particle is 6.87 MeV.

(a) What is the stopping power ($-dE/dx$) of water for the ^{211}At alpha?

From Figure 5.6 (p. 121 in Turner), the stopping power of water for the 6.87 MeV alpha particle emitted from ^{211}At is about **780 MeV cm^{-1}** .

(b) What is the range in water?

The relation for range of the two particles:

$$R_{\alpha}(\beta) = \frac{M}{Z^2} R_p(\beta) = R_p(\beta)$$

The velocity of the alpha particle and the proton is equal, but the ratio of kinetic energy of the two is given by:

$$\frac{T_{\alpha}}{T_p} = \frac{M_{\alpha}}{M_p} = 4$$

Solving for $T_p = 6.87/4 = 1.72$ MeV. This is the energy of the proton that is going to have the same range of that of the alpha particle with energy 6.87 MeV. Using interpolation for the range from Table 5.3 gives:

$$R = .0056 \frac{g}{cm^2} \cdot \frac{cm^3}{1g} = .0056 cm$$

-OR-

From Figure 5.7 (p. 128 in Turner), the range of the 6.87 MeV alpha in water is roughly **$7 \cdot 10^{-3}$ cm**.

4. (3 points) What thickness of Lucite (density 1.19 g cm^{-3}) would be required to completely shield the beta radiation from phosphorus-32 (1.7 MeV maximum energy).

Using equation 6.15 to calculate the range of these electrons:

$$R = 0.412(1.7)^{1.27 - .0954 \ln(1.7)} = 0.787 \frac{g}{cm^2} \cdot \frac{cm^3}{1.19g} = 0.661 cm \quad (6.15)$$

Hence at least 0.661 cm of Lucite is needed to completely shield against the beta radiation.

5). (4 points) ^{239}Pu with $E_{\alpha} = 5.16$ MeV

(a) Looking at figure 5.7, it is not possible to obtain a value for muscle for an alpha particle at this energy. However, the relation for range of the two particles:

$$R_{\alpha}(\beta) = \frac{M}{Z^2} R_p(\beta) = R_p(\beta)$$

The velocity of the alpha particle and the proton is equal, but the ratio of kinetic energy of the two is given by:

$$\frac{T_{\alpha}}{T_p} = \frac{M_{\alpha}}{M_p} = 4$$

Solving for $T_p = 5.16/4 = 1.29$ MeV. This is the energy of the proton that is going to have the same range of that of the alpha particle with energy 5.16 MeV. Using interpolation for the range from Table 5.3 gives:

$$R = .00345 \frac{g}{cm^2} \cdot \frac{cm^3}{1g} = .00345 cm$$

(b) In bone, the density is 1.9 g/cm³:

$$R = .00345 \frac{g}{cm^2} \cdot \frac{cm^3}{1.9g} = .0018 cm$$

(c) Looking at Figure 5.8, the range in air is approximately 2.5 cm

-OR-

You can use equation 5.45 and claim that air is incompressible from 20°-15°, and so:

$$R = 1.24E - 2.62 = 3.778 cm$$

6. (6 points) You have at your disposal an accelerator capable of accelerating any ion up to a maximum energy of 500 MeV *per nucleon*. You want to do a cell irradiation experiment with $^{12}_6C$ and with protons, **both having the same stopping power** (-dE/dx). You can pick any stopping power you want, but both ions must be able to penetrate the cell target, i.e., have a range of > 10 micrometers in water. Is this experiment possible? Why or why not?

From the stopping power formula, the velocity is inversely proportional to the stopping power. Hence, the minimum stopping power will occur at the maximum energy available and this will also be the maximum range possible. Lets then calculate the minimum stopping power for ^{12}C . To simplify things lets use Table 5.2 to obtain β and $F(\beta)$.

The maximum energy for the carbon is then 500 MeV * 12 = 6000 MeV. Lets find the equivalent energy for a proton at the same velocity:

$$\frac{T_C}{T_p} = \frac{M_C}{M_p} = 12 \Rightarrow T_p = 500 MeV$$

Interpolating from Table 5.2 for a 500 MeV proton gives $\beta^2 = 0.5696$ and $F(\beta) = 13.57$. This will be the same values for the carbon. Solving for the stopping power of carbon:

$$-\frac{dE}{dx} = \frac{0.170z^2}{\beta^2} [F(\beta) - 4.31] = \frac{0.170 \cdot 6^2}{0.5696} [13.57 - 4.31] = 99.5 \frac{MeV}{cm}$$

Now we want the stopping power of the proton to be equal to 100 MeV cm⁻¹ and looking at Table 5.3 we need to interpolate to find out what the range is for this stopping power. The range of the **proton is .022 cm**. The energy of the proton is 3.9 MeV. Lets find the range of the carbon. At this carbon energy of 6000 MeV, a proton of 500 MeV will have the same speed. The range of a 500 MeV proton from Table 5.3 is 115 cm. Solving for the range of this carbon:

$$R_C(\beta) = \frac{M}{Z^2} R_P(\beta) = \frac{12}{36} \cdot 115 = 38.3 \text{ cm}$$

Therefore the experiment is possible with a proton of roughly 4 MeV/nucleon and the carbon of a 500 MeV/nucleon will both have the same stopping powers and more than enough range to penetrate a cell target.

7. (4 points) An electron emerges normally from a 7-mm-thick Lucite slab (density 1.19 g cm^{-3}) with an energy of 1.2 MeV. What was its energy when it entered the slab?

We know that the positron emerges with energy of 1.2 MeV and can farther travel a distance of:

$$R = 0.412(1.2)^{1.27 - 0.0954 \ln(1.2)} = 0.5177 \frac{\text{g}}{\text{cm}^2} \cdot \frac{\text{cm}^3}{1.19 \text{ g}} = 0.435 \text{ cm} \quad (6.15)$$

The positron already went 7 mm in the slab, and so the total range that this positron has $0.435 \text{ cm} + .7 \text{ cm} = 1.135 \text{ cm}$ or a range of $R = 1.35 \text{ g/cm}^2$. Solving now 6.16:

$$\ln T = 6.63 - 3.24(3.29 - \ln R)^{1/2}; T = 2.79 \text{ MeV}$$

8. (6 points) Consider two particles: a 1.2 MeV proton and a 30.5 MeV alpha particle.

(a) Calculate the stopping powers ($-dE/dx$) of water for both particles.

For the proton, interpolating from Table 5.3 gives a stopping power of **240 MeV cm^{-1}** . Using Fig. 5.6, for the alpha particle, this gives a stopping power of about **240-250 MeV cm^{-1}** .

(b) What are the ranges of these particles in water?

For the proton, interpolating from Table 5.3 gives a range of **.003 cm** in water.

The relation for range of two particles going at the same speed:

$$R_\alpha(\beta) = \frac{M}{Z^2} R_p(\beta) = R_p(\beta)$$

The velocity of the alpha particle and the proton is equal, but the ratio of kinetic energy of the two is given by:

$$\frac{T_\alpha}{T_p} = \frac{M_\alpha}{M_p} = 4$$

Solving for $T_p = 30.5/4 = 7.63 \text{ MeV}$. This is the energy of the proton that is going to have the same range of that of the alpha particle with energy 7.63 MeV. Using interpolation for the range from Table 5.3 gives **0.073 cm**.

- (c) Calculate the maximum energy transferred by each in a collision with an orbital electron (assume that the electron is free and at rest).

$$\text{Proton: } Q_{\max} = \frac{4mME}{(M+m)^2} = \frac{4(0.511 \text{ MeV})(938.27 \text{ MeV})(1.2 \text{ MeV})}{(938.27 \text{ MeV} + 0.511 \text{ MeV})^2}$$

$$Q_{\max}(\text{p}) = \mathbf{0.0026 \text{ MeV} = 2.6 \text{ keV}}$$

$$\text{Alpha: } Q_{\max} = \frac{4mME}{(M+m)^2} = \frac{4(0.511 \text{ MeV})(3727.4 \text{ MeV})(30.5 \text{ MeV})}{(3727.4 \text{ MeV} + 0.51 \text{ MeV})^2}$$

$$Q_{\max}(\alpha) = \mathbf{0.0167 \text{ MeV} = 16.7 \text{ keV}}$$

- (d) What are the ranges in water of these delta ray electrons?

Interpolating from Table 6.1 for range:

For the 2.6 keV electron: **0.000042 cm.**

For the 16.7 keV electron: **0.0007 cm**