

Questions are solved in Columns
 Matlab Code below

2/21/06

BE.320 Problem Set #1

1. $\Delta G^\circ = RT \ln K_D$

$K_D = 10 \text{ nM}$

$\Delta G^\circ = 1.987 \frac{\text{cal}}{\text{mol}\cdot\text{K}} \cdot 298 \text{ K} \cdot \ln(10 \times 10^{-9})$
 $= -10.91 \frac{\text{kcal}}{\text{mol}} = -45.65 \frac{\text{kJ}}{\text{mol}}$

$K_D = 1 \text{ nM}$

$\Delta G^\circ = 1.987 \frac{\text{cal}}{\text{mol}\cdot\text{K}} \cdot 298 \text{ K} \cdot \ln(1 \times 10^{-9})$
 $= -12.27 \frac{\text{kcal}}{\text{mol}} = -51.34 \frac{\text{kJ}}{\text{mol}}$

$K_D = 100 \text{ pM}$

$\Delta G^\circ = 1.987 \frac{\text{cal}}{\text{mol}\cdot\text{K}} \cdot 298 \text{ K} \cdot \ln(100 \times 10^{-12})$
 $= -13.63 \frac{\text{kcal}}{\text{mol}} = -57.03 \frac{\text{kJ}}{\text{mol}}$

2. $U_{\text{elec}} = \frac{q_1 q_2}{4\pi\epsilon\epsilon_0 r_{12}}$

$r_{12} = 5 \times 10^{-10} \text{ m}$

$\epsilon_{\text{vacuum}} = \epsilon_0 = 8.854 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$

$\epsilon_{\text{protein interior}} = 4\epsilon_0 = 3.5416 \times 10^{-11} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$

$\epsilon_{\text{H}_2\text{O}} = 80\epsilon_0 = 7.0832 \times 10^{-10} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$

Unmodified state

$U_{\text{vacuum}} = \frac{(1)(-1)(1.602 \times 10^{-19})^2}{4\pi(8.854 \times 10^{-12})(5 \times 10^{-10})}$
 $= -4.613 \times 10^{-19} \text{ J}$

$= -1.103 \times 10^{-14} \text{ cal}$

$U_{\text{protein interior}} = \frac{(1)(-1)(1.602 \times 10^{-19})^2}{4\pi(3.5416 \times 10^{-11})(5 \times 10^{-10})}$
 $= -1.153 \times 10^{-14} \text{ J}$

$= -2.756 \times 10^{-20} \text{ cal}$

$U_{\text{H}_2\text{O}} = \frac{(1)(-1)(1.602 \times 10^{-19})^2}{4\pi(7.0832 \times 10^{-10})(5 \times 10^{-10})}$
 $= -5.766 \times 10^{-21} \text{ J}$

$= -1.378 \times 10^{-21} \text{ cal}$

Modified State

$U = \frac{q(1)}{4\pi\epsilon(5 \times 10^{-10})}$

$= 0 \text{ J}$ for all 3 cases

$= 0 \text{ cal}$

If the interaction was dipole-dipole, the calculation would change in that the energy falls as $\frac{1}{r^3}$ instead of $\frac{1}{r}$.

3. $\frac{-\Delta S^\circ}{R} \approx -53 \Rightarrow \Delta S^\circ = 53(1.987) = 105.3 \frac{\text{cal}}{\text{mol}}$
 $= 440.6 \frac{\text{J}}{\text{mol}}$

$\frac{\Delta H}{R} \approx \frac{-12.5 + 53}{2.45 \times 10^{-3}} \Rightarrow \Delta H = 10289.9(1.987) = 20.446 \frac{\text{kcal}}{\text{mol}\cdot\text{K}}$
 $= 85.546 \frac{\text{kJ}}{\text{mol}\cdot\text{K}}$

Graph and code below

4. $\Delta H(25) = -12 \frac{\text{kcal}}{\text{mol}}$

$\Delta H(40) = -13.5 \frac{\text{kcal}}{\text{mol}}$

$\Delta S(25) = -11.2 \frac{\text{cal}}{\text{mol}\cdot\text{K}}$

$\ln K_D = \frac{\Delta H^\circ}{RT} - \frac{\Delta S^\circ}{R}$

$\therefore K_D(25^\circ\text{C}) = e^{\frac{\Delta H^\circ}{RT} - \frac{\Delta S^\circ}{R}}$
 $= e^{\frac{-12000}{1.987 \cdot 298} - \frac{-11.2}{1.987}}$
 $= e^{-14.629}$
 $= 4.43 \times 10^{-7} \text{ M}$

$\Delta C_p = \frac{\Delta \Delta H}{\Delta T} = \frac{-13.5 + 12}{40 - 25} = \frac{-1.5}{15} = -0.1 \frac{\text{kcal}}{\text{mol}\cdot\text{K}}$

$\ln \left(\frac{K_D(T)}{K_D(T_0)} \right) = \frac{-\Delta H^\circ(T_0)}{R} \left(\frac{1}{T} - \frac{1}{T_0} \right) + \frac{\Delta C_p}{R} \left(\ln \left(\frac{T}{T_0} \right) + \frac{T_0}{T} - 1 \right)$

$\therefore \ln \left(\frac{K_D(25)}{K_D(40)} \right) = \frac{-\Delta H^\circ(40)}{R} \left(\frac{1}{298} - \frac{1}{313} \right) + \frac{\Delta C_p}{R} \left(\ln \left(\frac{298}{313} \right) + \frac{313}{298} - 1 \right)$

$\ln \left(\frac{4.43 \times 10^{-7}}{K_D(40)} \right) = \frac{13500}{1.987} \left(\frac{1}{298} - \frac{1}{313} \right) - \frac{100}{1.987} \left(\ln \left(\frac{298}{313} \right) + \frac{313}{298} - 1 \right)$

$\ln \left(\frac{4.43 \times 10^{-7}}{K_D(40)} \right) = 1.0309$

$\therefore K_D(40^\circ\text{C}) = \frac{4.43 \times 10^{-7}}{e^{1.0309}} = 1.580 \times 10^{-7} \text{ M}$

BE.320 Problem Set #1 (Continued)

5. a. Attached

b. The best interaction energy between the binding partners is predicted to be when the ligand has a +10 charge.

c. 1) solvation interactions between the unbound protein and water and the unbound ligand and water,

2) Van der Waals

3) Hydrophobic effect

4) entropy

5) hydrogen bonds

%BE.320

%Problem Set 1

close all;

clear all;

%Problem 3

Kd = [8e-10 1.2e-9 2e-9 3.7e-9 6.2e-9 1.2e-8 2e-8];

T = [55 50 45 40 35 30 25];

T = T + 273.15;

calculate_dh_ds(Kd,T)

%Problem 5

distance = cat(3, [2 2]*1e-10, [4 0]*1e-10, [0 0]*1e-10);

protein_charge = [1 -2] * 1.602e-19;

U = [];

for ligand_charge = -10:10

 U = [U interaction_energy([ligand_charge*1.602e-19],protein_charge,distance)];

end

ligand_charge = -10:10;

ligand_charge = ligand_charge * 1.602e-19;

figure(2)

plot(ligand_charge, U)

title('Interaction Energy Between Ligand and Protein')

xlabel('Ligand Charge (C)')

ylabel('Interaction Energy (J)')

%BE.320

%Problem Set 1

%Problem 3

function calculate_dh_ds(Kd, T)

x = 1./T;

y = log(Kd);

figure(1)

plot(x,y)

axis([0 3.4e-3 -55 -17.5])

title('Van't Hoff Plot')

xlabel('1/T (Kelvin⁻¹)')

ylabel('ln (K_D)')

end

%BE.320

%Problem Set 1

%Problem 5

```
function U = interaction_energy(ligand_charges, vector_charges, d)

    U = 0;

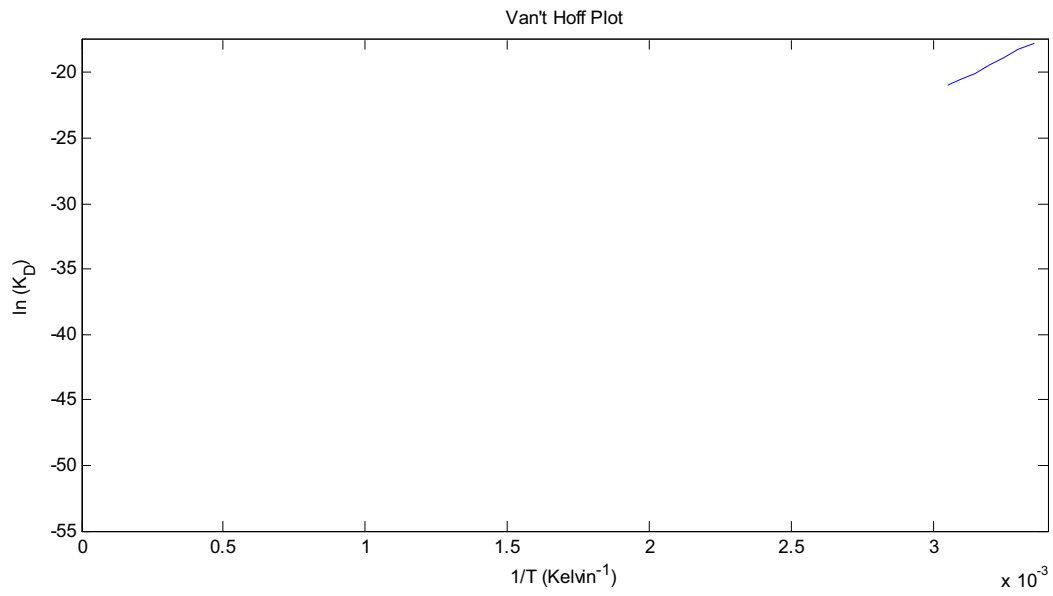
    %calculate ligand-vector interactions
    for i = 1:length(ligand_charges)
        for j = 1:length(vector_charges)
            U = U + (ligand_charges(i) * vector_charges(j)) / (4 * pi * 8.854e-12 * 4 * d
(i,j,1));
        end
    end

    %calculate vector-vector interactions
    for i = 1:length(vector_charges)
        for j = i+1:length(vector_charges)
            U = U + (vector_charges(i) * vector_charges(j)) / (4 * pi * 8.854e-12 * 4 * d
(i,j-1,2));
        end
    end

    %calculate ligand-ligand interactions
    for i = 1:length(ligand_charges)
        for j = i+1:length(ligand_charges)
            U = U + (ligand_charges(i) * ligand_charges(j)) / (4 * pi * 8.854e-12 * 4 * d
(i,j-1,3));
        end
    end

end
```

Problem 3 Graph:



Problem 5 Graph:

