6.012 Electronic Devices and Circuits

Formula Sheet for Hour Exam 2, Fall 2003

Parameter Values:

$q = 1.6x10^{19}\ Coul$

$_o = 8.854x10^{14}\ F/cm,\quad {}_{r,Si} = 11.7,\quad {}_{r,SiO_2} = 3.9$

$_{Si}\quad 10^{12}\ F/cm,\quad {}_{SiO_2}\quad 3.5x10^{13}\ F/cm$

$n_i[Si@R.T]\quad 10^{10}\ cm^{3}$

$kT/q\quad 0.025\ V;\quad (kT/q)\ln 10\quad 0.06\ V$

$1\quad m = 1x10^{4}\ cm$

Periodic Table:

III	IV	V
B	C	N
Al	Si	P
Ga	Ge	As
In	Sn	Sb

Drift/Diffusion:

Drift velocity: $\bar{s}_x = \pm\ {}_m E_x$

Conductivity: $= q(\ {}_e n + \ {}_h p)$

Diffusion flux: $F_m = \ D_m \frac{\partial C_m}{\partial x}$

Einstein relation: $\frac{D_m}{\ _m} = \frac{kT}{q}$

Electrostatics:

$$\frac{dE(x)}{dx} = \ (x) \qquad E(x) = \frac{1}{\ }\ \ (x)dx$$

$$\frac{d\ (x)}{dx} = E(x) \qquad (x) = \ \ E(x)dx$$

$$\frac{d^2\ (x)}{dx^2} = \ (x) \qquad (x) = \ \frac{1}{\ }\ \ (x)dxdx$$

The Five Basic Equations:

Electron concentration: $\frac{\partial n(x,t)}{\partial t}\ \ \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} = g_L(x,t)\ \ [n(x,t)\cdot p(x,t)\ \ n_i^2]r(T)$

Hole concentraton: $\frac{\partial p(x,t)}{\partial t} + \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = g_L(x,t)\ \ [n(x,t)\cdot p(x,t)\ \ n_i^2]r(T)$

Electron current density: $J_e(x,t) = q\ \ {}_e n(x,t)E(x,t) + qD_e\frac{\partial n(x,t)}{\partial x}$

Hole current density: $J_h(x,t) = q\ \ {}_h p(x,t)E(x,t)\ \ qD_h\frac{\partial p(x,t)}{\partial x}$

Poisson's equation: $\frac{\partial E(x,t)}{\partial x} = \ \frac{q}{\ }[p(x,t)\ \ n(x,t) + N_d^+(x)\ \ N_a(x)]$

Uniform doping, full ionization, TE

n - type, $N_d >> N_a$

$$n_o\ \ N_d\ \ N_a \equiv N_D,\quad p_o = n_i^2/n_o,\quad {}_n = \frac{kT}{q}\ln\frac{N_D}{n_i}$$

p - type, $N_a >> N_d$

$$p_o\ \ N_a\ \ N_d \equiv N_A,\quad n_o = n_i^2/p_o,\quad {}_p = \ \frac{kT}{q}\ln\frac{N_A}{n_i}$$

Uniform optical excitation, uniform doping

$$n = n_o + n' \qquad p = p_o + p' \qquad n' = p' \qquad \frac{dn'}{dt} = g_l(t)\ \ (p_o + n_o + n')n'r$$

Low level injection, $n', p' << p_o + n_o$: $\frac{dn'}{dt} + \frac{n'}{\ _{min}} = g_l(t)$ with $_{min}\ \ (p_o r)^{1}$

Flow problems (uniformly doped quasineutral regions with quasi-static excitation and low level injection; p-type example):

$$\text{Minority carrier excess:} \quad \frac{d^2 n'(x)}{dx^2} \quad \frac{n'(x)}{L_e^2} = \quad \frac{1}{D_e} g_L(x) \qquad L_e \equiv \sqrt{D_e \ \ _e}$$

$$\text{Minority carrier current density:} \quad J_e(x) \quad qD_e \frac{dn'(t)}{dx}$$

$$\text{Majority carrier current density:} \quad J_h(x) = J_{Tot} \quad J_e(x)$$

$$\text{Electric field:} \quad E_x(x) \quad \frac{1}{q \ \ _h p_o} J_h(x) \quad \frac{D_h}{D_e} J_e(x)$$

$$\text{Majority carrier excess:} \quad p'(x) \quad n'(x) + \frac{}{q} \frac{dE_x(x)}{dx}$$

Non-uniformly doped semiconductor sample in thermal equilibrium

$$\frac{d^2 \ \ (x)}{dx^2} = \frac{q}{}\left\{ n_i \left[e^{q \ \ (x)/kT} \quad e^{\ q \ \ (x)/kT} \right] \quad \left[N_d(x) \quad N_a(x) \right] \right\}$$

$$n_o(x) = n_i e^{q \ \ (x)/kT}, \quad p_o(x) = n_i e^{\ q \ \ (x)/kT}, \quad p_o(x) n_o(x) = n_i^2$$

Depletion approximation for abrupt p-n junction:

$$(x) = \begin{matrix} 0 & \text{for} & x < \ \ x_p \\ qN_{Ap} & \text{for} & x_p < x < 0 \\ qN_{Dn} & \text{for} & 0 < x < x_n \\ 0 & \text{for} & x_n < x \end{matrix}$$

$$N_{Ap} x_p = N_{Dn} x_n$$

$$_b \equiv \ \ _n \quad \ \ _p = \frac{kT}{q} \ln \frac{N_{Dn} N_{Ap}}{n_i^2}$$

$$w(v_{AB}) = \sqrt{\frac{2 \ \ _{Si} \left(\ \ _b \quad v_{AB} \right)}{q} \frac{\left(N_{Ap} + N_{Dn} \right)}{N_{Ap} N_{Dn}}} \qquad \left| E_{pk} \right| = \sqrt{\frac{2q \left(\ \ _b \quad v_{AB} \right)}{\ \ _{Si}} \frac{N_{Ap} N_{Dn}}{\left(N_{Ap} + N_{Dn} \right)}}$$

$$q_{DP}(v_{AB}) = \quad AqN_{Ap} x_p (v_{AB}) = \quad A \sqrt{2q \ \ _{Si} \left(\ \ _b \quad v_{AB} \right) \frac{N_{Ap} N_{Dn}}{\left(N_{Ap} + N_{Dn} \right)}}$$

Ideal p-n junction diode i-v relation:

$$n(\text{-}x_p) = \frac{n_i^2}{N_{Ap}} e^{qv_{AB}/kT}, \quad n'(\text{-}x_p) = \frac{n_i^2}{N_{Ap}} \left(e^{qv_{AB}/kT} \quad 1 \right); \qquad p(x_n) = \frac{n_i^2}{N_{Dn}} e^{qv_{AB}/kT}, \quad p'(x_n) = \frac{n_i^2}{N_{Dn}} \left(e^{qv_{AB}/kT} \quad 1 \right)$$

$$i_D = Aqn_i^2 \left[\frac{D_h}{N_{Dn} w_{n,eff}} + \frac{D_e}{N_{Ap} w_{p,eff}} \right] \left[e^{qv_{AB}/kT} - 1 \right] \qquad w_{m,eff} = \begin{matrix} w_m \quad x_m & \text{if } L_m >> w_m \\ L_m \tanh\left[\left(w_m \quad x_m \right) / L_m \right] & \text{if } L_m \sim w_m \\ L_m & \text{if } L_m << w_m \end{matrix}$$

$$q_{QNR,p\text{-}side} = Aq \int_{-w_p}^{-x_p} n'(x) dx, \qquad q_{QNR,n\text{-}side} = Aq \int_{x_n}^{w_n} p'(x) dx, \qquad \text{Note: } p'(x) \quad n'(x) \text{ in QNRs}$$

<u>Ebers-Moll Model for Bipolar Junction Transistor (BJT) characterisitics</u> (npn example; no base width modulation):

$$i_E = i_{hE} + i_{eE} = \quad Aqn_i^2 \quad \frac{D_h}{N_{DE} w_{E,eff}} + \frac{D_e}{N_{AB} w_{B,eff}} \left[e^{qV_{BE}/kT} \quad 1\right] \qquad\qquad {}_E \equiv \frac{i_{hE}}{i_{eE}} = \frac{D_h}{D_e} \cdot \frac{N_{AB}}{N_{DE}} \cdot \frac{w_{B,eff}}{w_{E,eff}}$$

$$i_C \quad i_{eE}(1 \quad {}_B) = \quad Aqn_i^2 \frac{D_e}{N_{AB} w_{B,eff}} \left[e^{qV_{BE}/kT} \quad 1\right](1 \quad {}_B) \qquad \text{with} \qquad {}_B \quad \frac{w_{B,eff}^2}{2D_e \quad {}_e} = \frac{w_{B,eff}^2}{2L_e^2}$$

$$i_B \quad i_{hE} + \ {}_B\, i_{eE} = i_{eE}(\ {}_E + \ {}_B) = \quad Aqn_i^2 \frac{D_e}{N_{AB} w_{B,eff}} \left[e^{qV_{BE}/kT} \quad 1\right](\ {}_E + \ {}_B) \qquad {}_F \equiv \frac{i_C}{i_B} = \frac{(1 \quad {}_B)}{(\ {}_E + \ {}_B)}$$

<u>Large Signal BJT Model in Forward Active Region (FAR)</u> (npn; with base width mod):

$$i_B(v_{BE}, v_{CE}) = I_{BS}\left[e^{qv_{BE}/kT} \quad 1\right] \quad \text{with} \quad I_{BS} \equiv \frac{1}{(\ {}_o + 1)} I_{ES} = \frac{1}{(\ {}_o + 1)} Aqn_i^2 \ \frac{D_{eB}}{w_{B,eff} N_{AB}} + \frac{D_{hE}}{w_{E,eff} N_{DE}}$$

$$i_C(v_{BE}, v_{CE}) = \quad {}_o[1 + \quad v_{CE}]\, i_B(v_{BE}, v_{CE}) = \quad {}_o I_{BS}\left[e^{qv_{BE}/kT} \quad 1\right][1 + \quad v_{CE}]$$

$$\text{Break-point model:} \quad v_{BE,on} \quad 0.6V, \quad v_{CE,sat} \quad 0.2V$$

<u>MOS Capacitor</u>:

$$\text{Flat-band voltage:} \quad V_{FB} \equiv v_{GB} \ \text{ at which} \quad (0) = \quad {}_p \ {}_{Si}$$

$$V_{FB} = \quad {}_p \ {}_{Si} \quad {}_m$$

$$\text{Threshold voltage:} \quad V_T \equiv v_{GC} \ \text{ at which} \quad (0) = \quad {}_p \ {}_{Si} + v_{BC}$$

$$V_T(v_{BC}) = V_{FB} \quad 2 \ {}_p \ {}_{Si} + \frac{1}{C_{ox}^*}\left\{2 \ {}_{Si}\, qN_A \left[\left|2 \ {}_p \ {}_{Si}\right| \quad v_{BC}\right]\right\}^{1/2} \qquad x_{DT}(v_{BC}) = \sqrt{\frac{2 \ {}_{Si}\left[\left|2 \ {}_p \ {}_{Si}\right| \quad v_{BC}\right]}{qN_A}}$$

$$\text{Inversion layer sheet charge density:} \quad q_N^* = \quad C_{ox}^*[v_{GC} \quad V_T(v_{BC})]$$

$$\text{Accumulation layer sheet charge density:} \quad q_P^* = \quad C_{ox}^*[v_{GB} \quad V_{FB}(v_{BC})]$$

<u>Gradual Channel Approx. for MOSFET characteristics</u> (n-channel; no channel length mod.):

$$\text{Valid for } v_{BS} \quad 0, \text{ and } \ v_{DS} \geq 0:$$

$$i_G(v_{GS}, v_{DS}, v_{BS}) = 0 \quad \text{and} \quad i_B(v_{GS}, v_{DS}, v_{BS}) = 0$$

$$0 \qquad \text{for} \quad \frac{1}{\ }[v_{GS} \quad V_T(v_{BS})] < 0 < v_{DS}$$

$$i_D(v_{GS}, v_{DS}, v_{BS}) = \qquad \frac{1}{2}\frac{W}{L} \ {}_e C_{ox}^*[v_{GS} \quad V_T(v_{BS})]^2 \qquad \text{for} \quad 0 < \frac{1}{\ }[v_{GS} \quad V_T(v_{BS})] < v_{DS}$$

$$\frac{W}{L} \ {}_e C_{ox}^* \ v_{GS} \quad V_T(v_{BS}) \quad \frac{v_{DS}}{2} \quad v_{DS} \qquad \text{for} \quad 0 < v_{DS} < \frac{1}{\ }[v_{GS} \quad V_T(v_{BS})]$$

$$\text{with} \quad V_T(v_{BS}) \equiv V_{FB} \quad 2 \ {}_p \ {}_{Si} + \frac{1}{C_{ox}^*}\left\{2 \ {}_{Si} qN_A \left[\left|2 \ {}_p \ {}_{Si}\right| \quad v_{BS}\right]\right\}^{1/2}$$

$$\equiv 1 + \frac{1}{C_{ox}^*} \ \frac{{}_{Si} qN_A}{2\left[\left|2 \ {}_p \ {}_{Si}\right| \quad v_{BS}\right]}^{1/2} \qquad C_{ox}^* \equiv \frac{{}_{ox}}{t_{ox}}$$

Large Signal MOSFET Model in Saturation (FAR) (n-channel; with base width mod.):

$$i_G(v_{GS}, v_{DS}, v_{BS}) \;=\; 0 \qquad\qquad i_B(v_{GS}, v_{DS}, v_{BS}) \quad\; 0$$

$$i_D(v_{GS}, v_{DS}, v_{BS}) \;=\; \frac{K}{2}\left[v_{GS} \quad V_T(v_{BS})\right]^2 [1+ \quad v_{DS}]$$

$$\text{with} \quad K \equiv \frac{W}{L} \;\; {}_e C_{ox}^* \quad \text{and} \quad V_T = V_{FB} \quad 2 \;\; {}_{p\;\;Si} + \frac{1}{C_{ox}^*}\left\{2 \;\; {}_{Si} q N_A \left[\left|2 \;\; {}_{p\;\;Si}\right| \quad v_{BS}\right]\right\}^{1/2}$$

Small signal linear equivalent circuits:

p-n Diode

$$g_d = \frac{q}{kT} I_{BS}\, e^{qV_{AB}/kT} \qquad \frac{q I_D}{kT}$$

$$C_d = C_{dp} + C_{df}, \text{ where } \quad C_{dp}(V_{AB}) = A\sqrt{\frac{q \;\; {}_{Si} N_{Ap}}{2(\;\; {}_b \quad V_{AB})}} \quad \text{and}$$

$$C_{df}(V_{AB}) = \frac{q I_D}{kT}\frac{\left[w_p \quad x_p\right]^2}{2D_e} = g_d \;\; {}_d \quad \text{with} \quad {}_d \equiv \frac{\left[w_p \quad x_p\right]^2}{2D_e}$$

BJT (in FAR)

$$g_m \quad \frac{q|I_C|}{kT} \qquad\qquad g \;\; = \frac{g_m}{\;\; {}_o} \qquad\qquad g_o \quad |I_C| \quad \text{or} \quad \left|\frac{I_C}{V_A}\right|$$

$$\text{C} \;\; = g_m \;\; {}_b + \text{ B - E depletion capacitance, where } \quad {}_b = \frac{w_B^2}{2D_{\substack{minority\\ .in\ base}}}$$

$$C \;\; = \text{ B - C depletion capacitance}$$

MOSFET (in saturation)

$$g_m \quad \sqrt{2K\,|I_D|} \qquad\qquad g_o \quad |I_D| \quad \text{or} \quad \left|\frac{I_D}{V_A}\right|$$

$$g_{mb} = \quad g_m = \quad \sqrt{2K\,|I_D|} \qquad \text{with} \qquad = \frac{1}{C_{ox}^*}\sqrt{\frac{{}_{Si} q N_A}{\left|q \;\; {}_p\right| \quad V_{BS}}}$$

$$C_{gs} = \frac{2}{3} W L C_{ox}^* \qquad\qquad C_{sb}, C_{gb}, C_{db}: \;\text{ depletion region capacitances}$$

$$C_{gd} = W\, C_{gd}^* \quad \text{where } C_{gd}^* \text{ is the gate - to - drain fringing and overlap}$$

capacitance per unit gate width