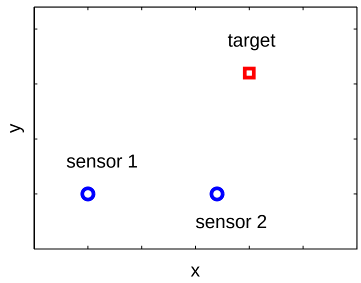
Massachusetts Institute of Technology
Subject 2.017 Design of Systems for Uncertain Environments

# DESIGN WORK 2

Assigned 16 February 2006
Due 23 February 2006

1. **Positioning using omnidirectional ranging; two-dimensional case.**

   (a) Two sensors, denoted 1 and 2 and located at different locations in the $x - y$ plane, make a range measurement to a target $t$. Let the range from sensor 1 to the target be $r_1$ and let the range from sensor 2 to the target be $r_2$; we call the sensor locations $[x_1, y_1]$ and $[x_2, y_2]$, and the target location $[x_t, y_t]$.

   What are the two equations describing the target's $[x_t, y_t]$ position in the horizontal plane, based on the range measurements from the two devices?

   (b) Sketch these constraints for the fixed sensor locations $x_1 = 0$, $y_1 = 0$, $x_2 = 1$, and $y_2 = 0$, for two different target locations a) $x_t = 2, y_t = 0$, b) $x_t = 0.5, y_t = 0.5$. This is a total of four circles to draw.

   (c) Consider your two constraints for a given target location; there are two unknowns $[x_t, y_t]$, so we ought to be able to solve for them. First, solve for $x_t$ by subtracting the two constraint equations, such that the $x_t^2$ and $y_t^2$ terms go away; be sure you take advantage of the fact that $x_1 = y_1 = y_2 = 0$! This will let you derive a clean expression for $x_t$. Then, put this value for $x_t$ into the constraint equation for sensor 1, and solve for $y_t$. There will be two solutions for $y_t$, because this array can't distinguish which side the target is on.

   (d) Now we model the whole thing in a program; consider the first target location. You are to calculate the two ranges that perfect sensors would give, perturb each of the perfect ranges by `0.1*randn` (a Gaussian random number with zero mean and unity variance) to simulate some sensor noise in a measurement, and then solve the real-world location problem, i.e., estimate $x_t$ and $y_t$, using the measured ranges.

   The idea is to understand how the noisy range measurements affect the estimate of where the target is. To see this, make a hundred noisy measurements, and plot all the target estimates on an $x-y$ plot that also shows the true target location. *Note that when the noisy measurement makes the square root negative in the equation for $y_t$, you have to set $y_t = 0$.*

   (e) Repeat this procedure for the second target location.

   (f) I find that the second configuration is about four times better than the first, in terms of scatter in the estimated location. Explain in words what is happening with the geometry of your constraint circles and the sensor noise.

2. **Aliasing.** Consider sampling sinusoidal process: $x(t) = 0.9 \sin(2\pi f_o t)$.

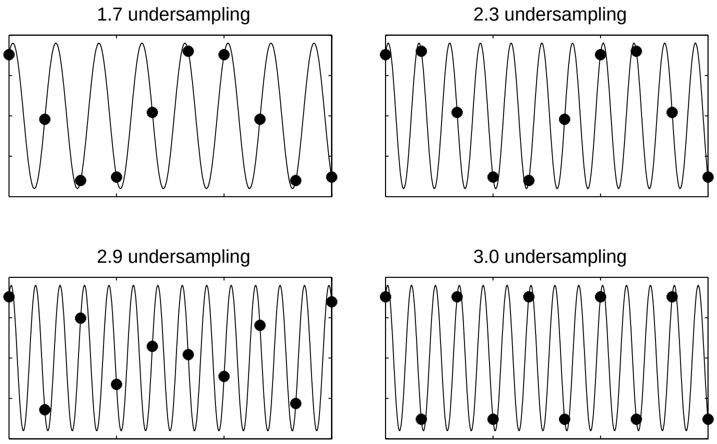
(a) When we sample the signal at frequency $f_s$ (Hz) or $\omega_s$ (rad/s), the Nyquist rate of $f_N = f_s/2$ is the maximum frequency the sampling can detect. See the figure for some examples; undersampling of 1.7 indicates that the process frequency exceeds the Nyquist rate by a factor of 1.7. What is the required sampling rate to detect $f_o$ = 20kHz? Extrapolating from the figure, is this sampling rate good enough to observe the process with reasonable fidelity?

(b) Suppose that you can only sample at $f_s = 5.5kHz$. What is the time step? For the time interval $t = [0\ 0.002]$, plot $x(t)$ sampled at a high enough rate that you can clearly see the sine wave, and overlay it with the signal sampled at $f_s$. What is the apparent (*aliased*) frequency of the sampled signal?

A simple rule to predict this aliased frequency is: decrement $f_o$ by $f_s$ enough times to get within the observable frequency range of $[-f_N,\ f_N]$. The absolute value of this result is the aliased frequency.

(c) Repeat the above analysis for $f_s = 6.5kHz$.

(d) Repeat the above analysis for $f_s = 7.5kHz$.




3. **Sea spectrum and vehicle pitch response.** The Bretschneider spectrum is given by

$$\begin{aligned}
S(\omega) &= \frac{A}{\omega^5} e^{-B/\omega^4}, \text{ where} \\
B &= 1.25\omega_m \\
\omega_m &= \text{ modal (or peak) frequency, rad/s} \\
A &= 4BE_S \\
E_S &= H_{1/3}^2/16.
\end{aligned}$$

(a) Make a plot of this spectrum for about one hundred frequencies from zero to 4 rad/s, with modal frequency $\omega_m = 6$ rad/s, and significant wave height $H_{1/3} = 0.90$m.

(b) Confirm that the area under the spectrum is equal to $E_S$, by making a numerical integration. You can take this $E_S$ to double-check $H_{1/3}$.

(c) Using the fact that $S(\omega_i)\delta\omega = a_i^2/2$, calculate a set of amplitudes $a_i$ that go with your frequency vector above. Then construct a time-domain signal by summing all of these amplitudes multiplied by sinusoids at the various frequencies; each frequency component needs to have a static, random phase. Make ten minutes worth of this wave-like data, using a sampling period of 0.1 seconds, and show a plot, with the original $H_{1/3}$ maximum and minimum levels indicated.

(d) Now suppose that the transfer function that takes wave motion $\eta$ into vehicle pitch $\phi$ is

$$F(j\omega) = \frac{\phi(j\omega)}{\eta(j\omega)} = \frac{0.1j\omega + 0.3}{-\omega^2 + j\omega + 3}.$$

Recalling that the spectrum of the system response is $Y(\omega) = F(j\omega)F^*(j\omega)S(\omega)$, compute the parameter $E_Y$ from the area under $Y(\omega)$, and so estimate the "significant height" of the pitch motion. $F^*$ is the complex conjugate of $F$, and you can get it in Matlab with `conj(F)`. Outside of Matlab, to conjugate is to negate the imaginary part. Also, note that $F(j\omega)F^*(j\omega) = |F(j\omega)|^2$, a real number.

4. **Project priorities.** WORKING AS A GROUP, and from the listings that were part of Design Work 1, clarify and establish the priorities (the order) of your Challenge objectives/requirements, and identify how strongly all the uncertain topics you identified will impact them. I recommend a tabular or matrix format for this, but you can be creative. *One document* should be emailed to the instructors on 23 February.