

14.05 Intermediate Applied Macroeconomics Problem Set 2 Solutions

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Question 1 The Solow Model and “War, What is it Good For?” (Exam # 1 Fall 2004)

Consider a standard Solow model described by the following equations:

$$\begin{aligned} Y &= (1 - \tau) K^\alpha (AL)^{1-\alpha}, & 0 < \alpha < 1 & \quad (1) \\ \dot{K} &= sY, \quad \frac{\dot{L}}{L} = 0, \quad \frac{\dot{A}}{A} = g \end{aligned}$$

Note: τ is a government tax on output, and all tax revenue collected by the government is spent on an ongoing war that does not contribute to either output or capital stock of the economy.

- (a) Define capital per effective worker as $k = K/AL$, and derive an expression for its evolution over time: dk/dt .

Answer

$$\begin{aligned} \dot{k} &= \frac{\dot{K}(AL) - K(\dot{A}L + \dot{L}A)}{(AL)^2} \\ \dot{k} &= \frac{\dot{K}}{AL} - k \left(\frac{\dot{A}}{A} + \frac{\dot{L}}{L} \right) \\ \dot{k} &= sy - gk \\ \dot{k} &= (1 - \tau) sk^\alpha - gk. \end{aligned}$$

- (b) Derive the long run equilibrium level of capital per effective worker, k^* , and the steady state output per-effective-worker, y^* , where $y = Y/AL$. Suppose the war takes an unfortunate turn for worse, and the government must increase the tax rate in order to buy more tanks. What will be the effect of increasing the tax on y^* ? [Hint: it will be easier to interpret y^* if g is in the denominator.]

Answer

$$\begin{aligned} \dot{k} &= (1 - \tau) sk^\alpha - gk = 0 \\ (1 - \tau) sk^{\alpha-1} &= g \\ k^* &= \left(\frac{(1 - \tau)s}{g} \right)^{\frac{1}{1-\alpha}} \\ y^* &= (1 - \tau)(k^*)^\alpha = (1 - \tau) \left(\frac{(1 - \tau)s}{g} \right)^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

If the government must increase the tax, this will decrease actual investment in the economy, and the steady state level of output per effective unit of labor, Y^* will decrease, i.e. $(dy^*/d\tau) < 0$.

- (c) Now suppose that the tax on output also hurts individual's incentives to invent new technologies. Specifically, assume that the growth rate of technology, g , is given by $g = b(1 - \tau)^{1/\alpha}$ where $b > 0$. What is the new steady state level of output per-effective-worker, y^* ? What is the effect of an increase in the tax on y^* now? Describe in words the two opposing effects of the tax on y^* . [Hint: Be sure to talk about the effect of the tax on 'actual investment' and 'break-even investment'.]

Answer

There is no need to resolve the model, and we can simply plug $g = b(1 - \tau)^{1/\alpha}$ into our solution for y^* from part (b) to get,

$$y^* = (1 - \tau) \left(\frac{(1 - \tau)s}{b(1 - \tau)^{\frac{1}{\alpha}}} \right)^{\frac{\alpha}{1 - \alpha}}$$

$$y^* = \left(\frac{s}{b} \right)^{\frac{\alpha}{1 - \alpha}}$$

The tax now has no effect on y^* . The reason for this is that the tax now has two opposing effects that exactly cancel each other out... The effects are: (i) The same we saw in part (b); by reducing actual investment, $(1 - \tau)sk^\alpha$, the tax reduces y^* . (ii) The other effect comes through the reduction in the growth rate of technology, g . With slower technology growth, less new capital is needed each period in order to maintain the same capital per effective unit of labor, and hence, k^* will be higher in the new steady state (as will be y^*). Graphically, this effect of the tax can be seen as a shift downward of the break-even investment curve.

- (d) Continue to assume $g = b(1 - \tau)^{1/\alpha}$, and recall that consumption is given by $C = (1 - s)Y$. What is the growth rate of consumption in the steady state? How will the increase in the tax rate affect the growth rate of consumption?

Answer

In steady state, $y = (Y/AL)$ is a constant. This implies that

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\dot{L}}{L}$$

$$\frac{\dot{Y}}{Y} = g + 0 = b(1 - \tau)^{1/\alpha}.$$

Then using $C = (1 - s)Y$, we have

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y}$$

$$\frac{\dot{C}}{C} = b(1 - \tau)^{1/\alpha}.$$

Thus, we can see that an increase in the tax rate will reduce the growth rate of consumption because of its negative effect on the growth rate of technology.

- (e) Suppose there is an upcoming election between Candidate A and Candidate B:
- (i) Candidate A says: “I will end the war, and I will eliminate the tax ...” So, if Candidate A is elected, the war will end and τ will fall to 0, such that $g = b$.
 - (ii) Candidate B says: “I will also end the war, but I will spend the tax revenues to improve education, etc.” So, if Candidate B is elected, the war will end, τ will remain unchanged, but the growth rate of technology will increase, such that $g = B(1 - \tau)^{1/\alpha}$ where $B > b$.

What will be the new steady state growth rate of consumption under each candidate’s plan? Under what condition will it be optimal for citizens to vote for Candidate B under the assumption that individuals only care about overall consumption growth? Explain in 1 or 2 sentences.

Answer

From part (d), we know that the growth rate of consumption simply equals the growth rate of technology in this economy. Therefore, under candidate A when $\tau = 0$, $g = b$, and hence the growth rate of consumption in the new steady state is simply equal to b . Under candidate B, the growth rate of consumption will be $g = B(1 - \tau)^{1/\alpha}$. It will be optimal for citizens to vote for candidate B, when $B(1 - \tau)^{1/\alpha} > b$.

Question 2 Endogenous Growth, Balanced Growth Path, and Scale Effects (Exam # 1 Fall 2004)

Consider the following model:

$$\begin{aligned} Y &= [(1 - a_K) K]^\alpha (AL)^{(1-\alpha)}, & 0 < \alpha < 1 \\ A &= B (a_K K)^\gamma, & 0 < a_K < 1, B > 0 \\ \dot{K} &= sY \\ \frac{\dot{L}}{L} &= n \end{aligned}$$

- (a) In 1-2 sentences, explain the two purposes capital, K , serves in this economy. What type of endogenous growth model is this?

Answer

Capital is used to produce output, Y , or to increase technology, A . A fraction a_K of capital is allocated towards improving technology. This is an R&D model. Notice that there is a sector that produces knowledge, thus activities are carried out in order to generate new knowledge, that is the reason why this falls into the R&D type of models (within the family of new growth theory models).

- (b) Let g_Y , g_K , and g_A represent the growth rates of output (Y), capital (K), and technology (A). Derive an expression for each growth rate.

Answer

Using a bit of algebra and log differentiation, we have

$$\begin{aligned} g_Y &= \alpha g_K + (1 - \alpha)(g_A + n) \\ g_A &= \gamma g_K \\ g_K &= s \left(\frac{Y}{K} \right) = s (1 - a_K)^\alpha K^{\alpha-1} (AL)^{1-\alpha} = s (1 - a_K)^\alpha K^{\alpha-1} (B (a_K K)^\gamma L)^{1-\alpha} \\ g_K &= s (1 - a_K)^\alpha B^{1-\alpha} a_K^{\gamma(1-\alpha)} L^{1-\alpha} K^{(1-\alpha)(\gamma-1)} \end{aligned}$$

- (c) Find an expression for $\frac{\partial g_K}{\partial t}$ that is a quadratic function of only g_K and other constants.

Answer

$$\begin{aligned} \frac{\dot{g}_K}{g_K} &= (1 - \alpha)n - (1 - \alpha)(1 - \gamma)g_K \\ \dot{g}_K &= (1 - \alpha)ng_K - (1 - \alpha)(1 - \gamma)g_K^2 \end{aligned}$$

- (d) Assume $n > 0$. With this assumption, what must be true about γ in order for this economy to have a positive balanced growth path (BGP)? In 1-2 sentences explain the intuition behind this restriction on γ .

Answer

In order to have a BGP, g_K , g_Y , and g_A must each be constant. From our equations, we see that g_Y and g_A will both be constant so long as g_K is. Therefore, we just need to ensure that $\dot{g}_K = (dg_K/dt) = 0$.

When $n > 0$, it is only possible for $\dot{g}_K = 0$ (at a positive value for g_K) when $\gamma < 1$. This is the necessary condition for a BGP. The intuition for this is as follows: A higher γ means that capital devoted to R&D has a larger impact on increasing technology. But, if γ is too high, the increases in the capital stock will cause even larger increases in the level of technology, and the growth rate of the economy will explode.

- (e) What is g_Y along the BGP of the economy $n > 0$? Let $y = Y/L$ represent output-per-capita. What is the growth rate of output-per-capita along the BGP? How does an increase in population growth rate, n , affect the growth rate of output-per-capita? In 1-2 sentences, explain the intuition behind why we have this population “scale effect” on growth.

Answer

The BGP occurs when $\dot{g}_K = 0$. From part (b), this implies that

$$\begin{aligned} g_K &= \frac{n}{1-\gamma} \\ g_A &= \frac{g\gamma}{1-\gamma}. \end{aligned}$$

Plugging these into our equation for g_Y in part (b), we have

$$g_Y = \frac{n\gamma}{1-\gamma} + n = \frac{n}{1-\gamma}.$$

Thus, the growth rate of output per capita is

$$g_y = g_Y - n = \frac{n\gamma}{1-\gamma}$$

An increase in the population growth rate increases the growth rate of output per capita because more workers \rightarrow more output \rightarrow increased capital accumulation \rightarrow more technology \rightarrow more output.

Question 3 Essays Based on the Assigned Readings

Be concise, go straight to the point unless explicitly required to link different papers. Write less than 10 lines per question.

Long answers makes it harder for the grader to find the right arguments, thus restrain yourself from using too many words.

- (a) According to Acemoglu, Johnson and Robinson (“Colonial Origins...”), what institutions differed across colonial countries, why did they differ, and how did this matter for growth?

Answer

According to Acemoglu, Johnson, and Robinson, the institutions concerning the protection of property rights and checks and balances on the government’s ability to extort its citizens vary across colonial countries. i.e. Some of the colonial countries have “extractive institutions” and others have more European style institutions that protect property rights and provide checks and balances on the government.

Extractive (or poor) institutions were implemented in areas where mortality rates were very high for Europeans (because of diseases, etc.), and colonizers could not settle. These “extractive institutions” did not promote growth.

In regions where mortality rates were low, however, European were able to settle, and these settlers implemented institutions similar to those in their home country. By protecting property rights and placing checks on the government’s ability to extort, these regions were more conducive to economic growth.

- (b) How has the world distribution of income-per-capita evolved the latter half of the 20th century according to Charles Jones? In particular, do we see convergence or divergence in income-per-capita between countries? Or, do we see both? What does the Solow model predict we should see for countries that share the same investment rates, depreciation, and population growth rate?

Answer

Jones finds that the world distribution of per-capita income moved from a single peaked distribution in 1960 to a double peaked distribution by 1988. Countries with a relatively more income per capita in 1960 converged upwards towards the per-capita level of the U.S., but countries with less per-capita income in 1960 actually fell further behind. In other words, we see convergence at the top of income distribution, and divergence at the bottom.

If countries share the same characteristics, such as investment rates, depreciation rates, and population growth rates, the Solow model does indeed predict that we should see convergence. Poorer countries are simply further away from the shared steady state, and hence they should be growing faster than the rich countries.

- (c) What is the main structural difference Jeffrey Sachs points out between the economies of East Asia and those of Eastern Europe and the former Soviet Union (EEFSU)? According to Sachs, what is the link between this difference and the need for “shock therapy” in EEFSU compared to East Asia?

Answer

A main difference noted by Jeffrey Sachs is that the East Asian economies had a larger amount of labor in the agricultural sector of the economy, and less labor in the state owned enterprises (SOEs) than many EEFSU countries.

Moreover, workers in the agricultural sector of East Asian economies were implicitly taxed and were very willing to work in a new, more productive private sector, while workers in the agricultural sector of EEFSU countries were implicitly subsidized and less willing to leave agriculture.

Because of this, East Asian countries could more easily move labor into the new, more productive private sector without dramatic changes in the organization of the SOEs. In the EEFSU, however, there was not a large amount of labor that could be easily moved into the new private sector because a large portion of the labor force was employed in SOEs and had very little incentive to move into the more productive private sector. Thus, without adopting a “shock” approach of dramatically cutting back the SOEs, the countries of the EEFSU would be unable to move labor into the more productive private sector.

- (d) Svejnar presents an analysis of the performance of the transition economies. In his analysis he looks at the evolution of the Gross Domestic Product (GDP), among many other variables. Why do we see differences between these countries? Relate Svejnar’s discussion to the new growth theories we have discussed.

Answer

The main difference between these countries is in the implementation of Type II reforms (according to Svejnar’s definition). The author claims that countries that manage to implement more reforms of this type actually performed better.

From a growth theory point of view, the Type II reforms can be associated to the introduction of a market-oriented economy and the creation of an environment that provides the right incentives for an efficient allocation of resources and technological progress (Svejnar does not mention it but here innovation and technology adoption play a significant role). Then, growth is higher because these economies achieve a higher level of efficiency and can use their resources better, but also innovation and adoption get a bigger push, which is the main point emphasized in new growth theory. In this framework, Type I reforms provide a basic benchmark, but Type II reforms lead to the build-up of economic factors more friendly to long term growth.

Although the author recognizes that geography did play some role -countries located further west shared more common things with Western Europe countries thus facilitating the assimilation and integration process-, he also claims that this can not be the reason for the differences.