

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
**15.053 – Optimization Methods in Management Science (Spring 2007)**

Problem Set 11, Due December 3<sup>rd</sup> 2013  
You will need 0 points out of 0 to receive a grade of 0

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**Prologue:**

Nooz, feeling he was promised an equal starring role in the course, and in the end taking a backseat to Ollie and Tim, decides to leave the course to pursue a few “real world jobs”. Will this sly fox make it in the real world and cease to exist as a 15.053 character or return to his role as our number one fox? In this problem set, we follow the adventures of Nooz while learning about our final topic decision analysis.

**Problem 1:**

**Nooz Shakes the Steak**

Nooz’s Chicken Coop and Steak and Shake are two competing restaurants. Each must determine simultaneously whether to undertake small, medium, or large advertising campaigns. Nooz believes that it is equally likely that Steak and Shake will undertake a small, a medium, or a large advertising campaign. Given the actions chosen by each restaurant, Nooz’s profits are as shown below.

Nooz Chooses	Steak and Shake Chooses		
	Small	Medium	Large
Small	4,000	3,000	2,000
Medium	5,000	4,000	3,000
Large	6,000	5,000	1,500

Determine Nooz’s choice under the following decision criteria:

**Part A:**

maximin

**Part B:**

maximax

**Part C:**

minimax regret

**Part D:**

expected value (here, assume Steak and Shake's three choices are equally likely.)

**Problem 2:****Foxes and Fools**

Having lost the fast food battle; Nooz decides to start a new News station called Fox Nooz. The Fox Nooz station generates a profit of \$5 million on average for a new show that becomes a hit, and loses \$1 million on average for a show that is a failure. Of all shows that Fox Nooz is considering, they expect that 30% of them will be a hit and 70% will be a failure.

For the cost of \$500,000, the station can make a pilot that will indicate whether or not a show will be a hit or a failure. If the pilot is a hit, the series will be a hit with 100% probability. If the pilot is a failure, then with 100% probability it will be a failure. (In practice, pilots are not so reliable as indicators, and the numbers are chosen here to make the problem simpler.)

Fox Nooz is thinking of producing the new reality series entitled, 'Who Is the Biggest Fool: Bill Clinton or Turkey Tim?'. The show puts two Democrats against each other in a variety of clown-like challenges. Fox Nooz needs to decide whether or not to produce the show and whether or not to produce a pilot for the show.

**Part A:**

Set up a decision tree and solve the problem. What should Fox Nooz do, and what is their expected profit?

**Part B:**

The maximum value that they would be willing to pay for the pilot is the expected value of perfect information (EVPI). What is the maximum value that they would be willing to pay for the pilot in this situation? Note: it will be the value at which Fox Nooz is indifferent between making the pilot and not making the pilot.

### **Problem 3: Nooz of the Opera**

**The phantom problem strikes again.**

### **Problem 4:**

#### **Nooz vs. Serena Williams**

**Please note: The context of the problem is prior to 2002 when Serena cared about tennis more than acting, designing, drinking, complaining, crying, eating, shoe collecting, insect watching, acting, listening to Jessica Simpson music, 15.053 POD Cast Watching, and Ollie Admiring. Revised Note: As of January 2007 This problem is again relevant.**

In many decision tree problems, the decision maker's goal is to maximize the probability of a favorable event occurring. To incorporate this goal into a decision tree, simply assign a reward of 1 to any terminal branch that results in the favorable event occurring, and a reward of 0 to any terminal branch that results in the favorable event not occurring. Then maximizing the expected reward is the same as maximizing the probability that the favorable event will occur.

Use this idea to solve the following problem:

After the shocking failure of the show "Who is the biggest fool?" Nooz decides to pursue his dream of becoming a professional tennis player. With great success he finds himself in the world tennis championships. Serena Williams is playing Nooz a single point for the world tennis championship. She has won the coin toss and elected to serve. If she tries a hard serve, her probability of getting the serve into play is .60. Given that the hard serve is in play, she has a .60 chance of winning the point. If she tries a soft serve, her probability of getting the serve into play is .90, but if the soft serve is in play, her probability of winning the point is only .50. To maximize her probability of winning the point, what should Serena do? What is the probability she will win, if she makes an optimal choice? (Recall that in tennis, players have two chances at a serve; if the first serve is out of play, then Serena can serve again. She can make different decisions on each of the serves about whether to play hard or soft.)

### **Problem 5:**

#### **The Nooz Vendor Problem**

After missing an easy job and losing the match, Nooz decides to start a computer company. Nooz needs to decide on the number of laptops to sell during the fall season. Assume that he will not be able to sell any computers once the fall rush is over — that is, any leftover laptops will be worth nothing.

Nooz buys a laptop for \$600 and sells it for \$1350. He wants to maximize his profit. The overstocking cost  $c_o$  of buying one computer too many is \$600. The understocking cost  $c_u$  of Nooz buying one computer too few is  $\$1350 - 600 = 750$ . (That is, it is the profit that could have been made, but wasn't.)

The following table shows the cumulative probability function for a Normally distributed random variable with mean 60 and standard deviation of 8, and of a uniformly distributed random variable between 35 and 85, rounded off to integer values.

P(x<=C)	N(60,8)	U(35,85)	P(x<=C)	N(60,8)	U(35,85)	P(x<=C)	N(60,8)	U(35,85)
25	0.0000	0	50	0.1175	0.31	75	0.9737	0.81
26	0.0000	0	51	0.1440	0.33	76	0.9804	0.83
27	0.0000	0	52	0.1743	0.35	77	0.9856	0.85
28	0.0000	0	53	0.2083	0.37	78	0.9896	0.87
29	0.0001	0	54	0.2459	0.39	79	0.9926	0.89
30	0.0001	0	55	0.2869	0.41	80	0.9948	0.91
31	0.0002	0	56	0.3309	0.43	81	0.9964	0.93
32	0.0003	0	57	0.3773	0.45	82	0.9975	0.95
33	0.0005	0	58	0.4256	0.47	83	0.9983	0.97
34	0.0007	0	59	0.4751	0.49	84	0.9989	0.99
35	0.0011	0.01	60	0.5249	0.51	85	0.9993	1
36	0.0017	0.03	61	0.5744	0.53	86	0.9995	1
37	0.0025	0.05	62	0.6227	0.55	87	0.9997	1
38	0.0036	0.07	63	0.6691	0.57	88	0.9998	1
39	0.0052	0.09	64	0.7131	0.59	89	0.9999	1
40	0.0074	0.11	65	0.7541	0.61	90	0.9999	1
41	0.0104	0.13	66	0.7917	0.63	91	1.0000	1
42	0.0144	0.15	67	0.8257	0.65	92	1.0000	1
43	0.0196	0.17	68	0.8560	0.67	93	1.0000	1
44	0.0263	0.19	69	0.8825	0.69	94	1.0000	1
45	0.0350	0.21	70	0.9053	0.71	95	1.0000	1
46	0.0458	0.23	71	0.9247	0.73	96	1.0000	1
47	0.0591	0.25	72	0.9409	0.75	97	1.0000	1
48	0.0753	0.27	73	0.9542	0.77	98	1.0000	1
49	0.0947	0.29	74	0.9650	0.79	99	1.0000	1
						100	1.0000	1

**Part A:**

Maximize the Nooz's profit if you assume that the integral demand is uniformly distributed between 35 and 85. How many laptops should the Nooz purchase? Note: for  $35 \leq x \leq 85$ ,  $\text{Prob}(\text{Demand} \leq x) = (x-34)/51$ .

**Part B:**

Maximize Nooz's profit if you assume that the integral demand is based on a normal distribution with mean 60 and standard deviation of 8. In this case,  $\text{Prob}(\text{Demand} \leq x)$  is given in the table. How many laptops should Nooz purchase?

### **Problem 6:**

#### **Risky Nooz on Wall Street**

Unfortunately Nooz's, Laptop business was destroyed in a fire. He decides to start a commodities firm called, Nooz Notes. Nooz has acquired an option to buy 1,000 pounds of Welsh rabbit meat at \$50/lb. If he takes the option and Congress relaxes import quotas, he can sell the rabbit meat for \$80/lb. If he takes the option and Congress does not relax the import quotas, the company will lose \$10/lb. Nooz believes that there is a 40% chance that the government will relax the quota. He also has the option of waiting until Congress decides whether to relax the import quota. If he adopts this strategy, there is a 75% chance that some other broker will have already taken the option, and so he only has a 25% chance of being able to buy the 1000 lbs. Welsh Rabbit.

#### **Part A:**

If Nooz is risk-neutral, what should he do? Draw the decision tree. What is the expected profit associated with that decision?

#### **Part B:**

If Nooz's utility function for the change  $x$  in his assets is given by

$$u(x) = (10,000 + x)^5,$$

what should he do? Draw the decision tree. What is the expected utility associated with this decision? How many dollars does this translate to?

### **Problem 7:**

#### **The New Lottery**

After realizing that the commodity market is unstable, Nooz turns to the stable world of lotteries. Let  $L$  represent the following lottery. Nooz tosses a coin until it comes up heads. If the first heads is obtained on the  $n$ th toss of the coin, Nooz receive a payoff of  $\$2^n$ .

#### **Part A:**

If Nooz were a risk-neutral decision maker, what would be the certainty equivalent of  $L$ ?

#### **Part B:**

Does the answer from Part A make sense in the real world? What is strange about it?

**Part C:**

If Nooz's utility function for increasing wealth by  $x$  dollars is given by  $u(x) = x^{.5}$ , what would be the certainty equivalent of  $L$ ? (Your answer should be a number!!!)

**Epilogue:**

After losing all his money in the lottery, Nooz broke on the streets of Vegas, makes a call to the ever loyal Professor Orlin, saying what an error he made and the only righteous career for him is being a 15.053 character. Did professor Orlin take him back? Of course he did! Welcome home Nooz.