A. Physical constants and conversion factors

Quantity	Symbol	Value	Units
Atomic mass unit	amu	931.5 1.661×10^{-27}	MeV/c^2 kg
Electron mass	m_e	0.511 9.109×10^{-31}	MeV/c^2 kg
Proton mass	m_p	939 1.673×10^{-27}	MeV/c^2 kg
Elementary charge	q	1.602×10^{-19}	C
Planck's constant	h	6.626×10^{-34} 4.136×10^{-15}	J·s eV·s
	$\hbar=h/2\pi$	1.054×10^{-34} 6.583×10^{-16}	J·s eV·s
Speed of light in vacuum	c	2.998×10^{8}	m/s
Boltzmann's constant	k_B	1.381×10^{-23}	J/K
Permittivity of free space	ε_0	8.854×10^{-12}	$C^2N^{-1}m^{-2}$
Avogadro's number	N_A	6.022×10^{23}	mol^{-1}
Rydberg constant	R_∞	3.158×10^{15}	Hz

$$1eV = 1.602 \times 10^{-19}\ J$$
$$1\ Å = 10^{-10}\ m$$
$$hc = 12400\ eV\ Å$$
$$\hbar c = 1973\ eV\ Å$$

B. Selected formulae

Differential cross section for Rutherford scattering: $$\frac{d\sigma}{d\Omega} = \frac{(Z'Z)^2 q^4}{256\pi^2\varepsilon_0^2 E^2}\frac{1}{\sin^4(\theta/2)}$$

Compton shift $$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$$

Radius of allowed orbits in Bohr model of hydrogenic atom: $$r_n = \frac{n^2}{Z}a_0,$$

where $a_0 = \dfrac{4\pi\varepsilon_0\hbar^2}{q^2 m} = 0.529Å$ is the Bohr radius.

Ground-state energy of hydrogen: $$E_1 = -13.6eV$$

Solid angle of thin ring of width dθ at angle θ: $$d\Omega = 2\pi\sin\theta\, d\theta$$

Fourier transform: $$\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\psi(x)e^{-ikx}dx \qquad \psi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(k)e^{ikx}dk$$

Short problems (30 points)

Please compute numerical answers to two significant digits.

a) **Rutherford experiment** (10 points)

i) (5 points) Please explain in two sentences: What is the important feature in the outcome of the Rutherford experiment, and what can be inferred about the structure of an atom?

ii) (5 points) Estimate the incident energy (numerical value in J or MeV) at which the Rutherford scattering formula breaks down for scattering off silver (Z=47).

b) **Compton scattering** (10 points)

Assume that a photon is scattered by an electron initially at rest. Which photon scattering angle corresponds to the largest Compton shift and why? At what minimum photon energy can half of the photon energy be transferred onto the electron?

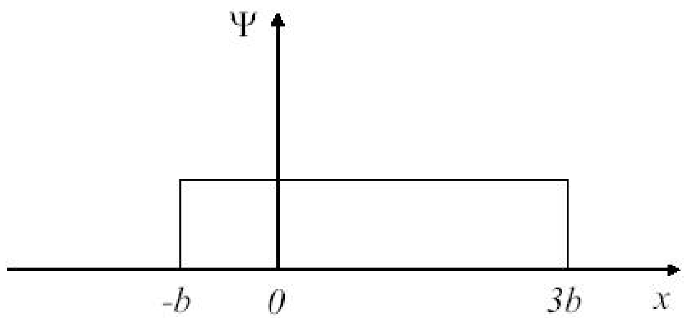
c) **X-rays** (10 points)

How large an electron acceleration voltage is necessary to produce X-rays with a wavelength of 1Å? If the work function of the metal that the electrons are incident upon is 5eV, how large is the fractional correction to the wavelength?

2. **Square wave function.** (30 points)

The state of a free particle is described by the following wave function:

$$\psi(x) = \begin{cases} 0 & \text{for } x < -b \\ A & \text{for } -b \le x \le 3b \\ 0 & \text{for } x > 3b \end{cases}$$



(a). (5 points) Find A using the normalization condition. (You may choose the phase convention such that A is real.)

(b). (5 points) What is the probability of finding the particle within the interval [0, b]?

(c). (10 points) Calculate $\langle x \rangle$ and $\langle x^2 \rangle$ for this state.

(d) (10 points) Calculate the momentum probability density.

3. **Heisenberg uncertainty** (25 points)

a) Early neutron model (10 points).

The neutron is an electrically neutral particle with a mass approximately equal to the proton mass. An early model considered the neutron to be an object where the electron is confined inside the proton. Assuming that the proton radius is $R = 10^{-15}$m, estimate the electron's kinetic energy due to Heisenberg uncertainty, and compare it to the neutron rest mass.

b) Energy spread of electron beam (15 points)

A monochromatic beam of electrons of energy E = 1 keV is incident on a shutter that opens for t=1ns. What is the fractional energy spread $\Delta v/v$ of the electron velocity v after the shutter? (Decide first whether to perform a relativistic or a nonrelativistic calculation.)

4. **Double slit experiment.** (15 points)

Electrons impinge on a double slit and form an interference pattern on a far-away screen with spatial period *s*. The slits have equal *width that is much smaller than the electrons' deBroglie wavelength*. The contrast *C* of the interference pattern is defined as $C = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$, where I_{max}, I_{max} are the maximum and minimum intensity on the screen.

a) (5 points) Assume that we have a way of changing the phase of the wavefunction at each of the slits without changing its amplitude. If we change the phase of the wavefunction at slit 1 by ϕ_1, and the phase of the wave function at slit 2 by ϕ_2, what happens to the electron interference pattern on the screen? (Position and contrast.) Explain your answer with a formula or a sentence.

b) (5 points) Now assume that we do not change the phases of the wavefunctions at the slits, but instead make slit 1 half as wide as slit 2. What happens to the interference pattern on the screen? (Position and contrast.) Explain your answer with a formula or a sentence.

c) (5 points) What happens to the interference pattern if we replace the electrons impinging on the double-slit by muons of the same energy? If the interference pattern changes, specify the change quantitatively. The mass of the muon is 207 times larger than that of the electron, the charge is the same. Both the electrons and the muons are assumed to be non-relativistic.