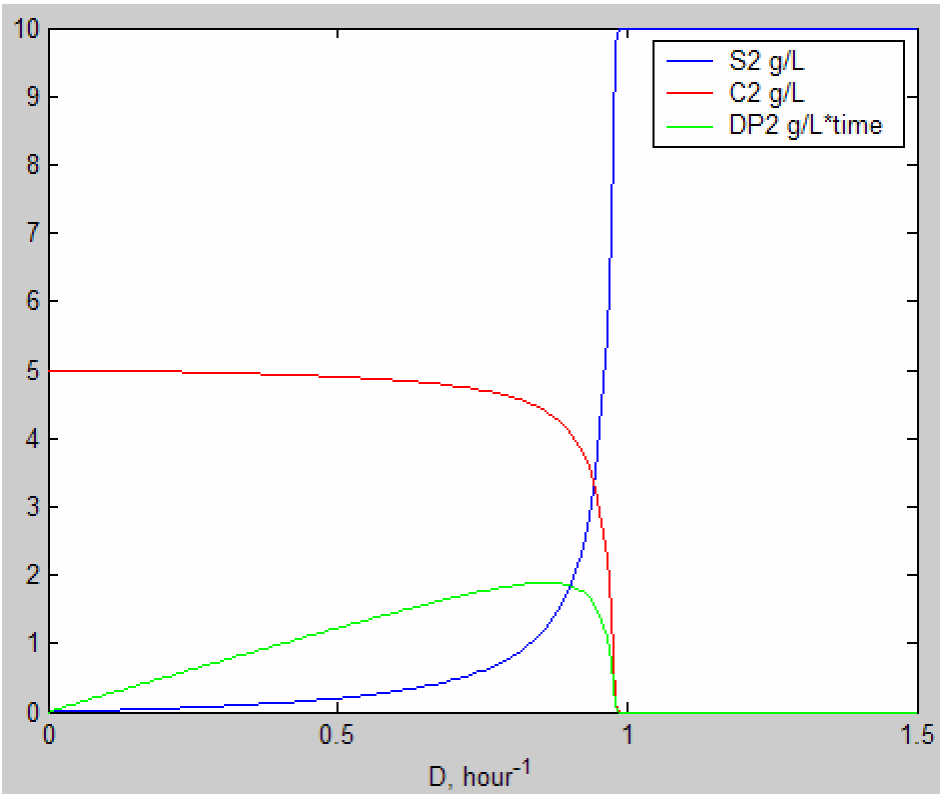
1.

The code is attached – this code makes the plot below and outputs that the maximum DP is 1.8858 g/L*hr and that the optimal D is 0.8593 hour^-1. These values will differ slightly if you used a different number of x-axis points than I did.



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%pset9 bioreactor problem

%constants
mu_max = 1.0; %hour^-1
Km = 0.2; %g/L
Sin = 10; %g/L
Ys = 0.5;
Yg = 0.5;

D = linspace(0,1.5,200); %hour^-1

for i=1:200
    S2(i) = (D(i)*Km)/(mu_max - D(i));
    C2(i) = Ys*(Sin-S2(i));
    DP2(i) = D(i)*Yg*C2(i);
    %constraints for mu_max < D and for physical concentrations
    if ((mu_max < D(i)) | (C2(i) < 0) | (DP2(i) < 0))
        S2(i) = Sin;
        C2(i) = 0;
        DP2(i) = 0;
    end
end

plot(D,S2,'b');
hold on
plot(D,C2,'r');
plot(D,DP2,'g');
xlabel('D, hour^{-1}');
legend('S2 g/L','C2 g/L','DP2 g/L*time');

maxDP2 = max(DP2)
temp = maxDP2-0.0001;
optimal_idx = find(DP2>temp);
optimal_D = D(optimal_idx)
```

2.

1st Stage

$$C(0) = C_0$$

$$S(0) = S_0$$

$$V\frac{dC}{dt} = V\mu_{\max}C$$

$$V\frac{dS}{dt} = -\frac{V\mu(s)C}{Y_S} = -\frac{V\mu_{\max}C}{Y_S}$$

$$V\frac{dP}{dt} = 0$$

2nd Stage

$$S(t_1) = S_C$$

$$C(t_1) = \text{constant}$$

$$V\frac{dC}{dt} = 0$$

$$V\frac{dS}{dt} = -Vk_{depletion}SC(t_1)$$

$$V\frac{dP}{dt} = k_{product}\frac{dS}{dt}$$

Stop

$$S(t_2) = S_F$$

1st Stage

$$C(0) = C_0$$

$$S(0) = S_0$$

$$V \frac{dC}{dt} = V \mu_{\max} C$$

$$\frac{1}{C} dC = \mu_{\max} dt$$

$$\ln C = \mu_{\max} t$$

$$C = C_0 e^{\mu_{\max} t}$$

$$V \frac{dS}{dt} = -\frac{V \mu_{\max} C}{Y_S}$$

$$dS = -\frac{\mu_{\max} C}{Y_S} dt = -\frac{\mu_{\max} C_0 e^{\mu_{\max} t}}{Y_S} dt$$

$$S = \frac{-C_0 e^{\mu_{\max} t}}{Y_S} + B_0$$

$$S_0 = \frac{-C_0 e^{\mu_{\max} 0}}{Y_S} + B_0$$

$$B_0 = S_0 + \frac{C_0}{Y_S}$$

$$S = \frac{-C_0 e^{\mu_{\max} t}}{Y_S} + S_0 + \frac{C_0}{Y_S}$$

2nd Stage

$$S(t_1) = S_C$$

$$S = \frac{-C_0 e^{\mu_{\max} t}}{Y_S} + S_0 + \frac{C_0}{Y_S}$$

$$S_C = \frac{-C_0 e^{\mu_{\max} t_1}}{Y_S} + S_0 + \frac{C_0}{Y_S}$$

$$S_C - S_0 - \frac{C_0}{Y_S} = \frac{-C_0 e^{\mu_{\max} t_1}}{Y_S}$$

$$S_0 - S_C + \frac{C_0}{Y_S} = \frac{C_0 e^{\mu_{\max} t_1}}{Y_S}$$

$$\left(S_0 - S_C + \frac{C_0}{Y_S} \right) \frac{Y_S}{C_0} = \left(\frac{\left(S_0 - S_C \right) Y_S}{C_0} + 1 \right) = e^{\mu_{\max} t_1}$$

$$\ln \left(\frac{\left(S_0 - S_C \right) Y_S}{C_0} + 1 \right) = \mu_{\max} t_1$$

$$t_1 = \frac{\ln \left(\frac{\left(S_0 - S_C \right) Y_S}{C_0} + 1 \right)}{\mu_{\max}}$$

$$C(t_1) = C_0 e^{\mu_{\max} t_1} = C_0 e^{\ln \left(\frac{\left(S_0 - S_C \right) Y_S}{C_0} + 1 \right)} = C_0 \left(\frac{\left(S_0 - S_C \right) Y_S}{C_0} + 1 \right) = \left(\left(S_0 - S_C \right) Y_S + C_0 \right)$$

$$V\frac{dC}{dt}=0$$

$$V\frac{dS}{dt}=-Vk_{depletion}SC(t_1)=-Vk_{depletion}S\left(\left(S_0-S_C\right)Y_S+C_0\right)$$

$$\frac{1}{S}dS=-k_{depletion}\left(\left(S_0-S_C\right)Y_S+C_0\right)dt$$

$$\ln S=-k_{depletion}\left(\left(S_0-S_C\right)Y_S+C_0\right)t$$

$$S=A_0e^{-k_{depletion}\left(\left(S_0-S_C\right)Y_S+C_0\right)t}$$

$$S(t_1)=S_C$$

$$S_C=A_0e^{-k_{depletion}\left(\left(S_0-S_C\right)Y_S+C_0\right)t_1}$$

$$A_0=\frac{S_C}{e^{-k_{depletion}\left(\left(S_0-S_C\right)Y_S+C_0\right)t_1}}$$

$$\therefore S=\frac{S_Ce^{-k_{depletion}\left(\left(S_0-S_C\right)Y_S+C_0\right)t}}{e^{-k_{depletion}\left(\left(S_0-S_C\right)Y_S+C_0\right)t_1}}$$

$$\frac{dP}{dt}=k_{product}\frac{dS}{dt}=-k_{product}k_{depletion}S\left(\left(S_0-S_C\right)Y_S+C_0\right)$$

$$dP=-k_{product}k_{depletion}S\left(\left(S_0-S_C\right)Y_S+C_0\right)dt$$

$$P=-k_{product}k_{depletion}S\left(\left(S_0-S_C\right)Y_S+C_0\right)t$$

Stop

$$S(t_2) = S_F$$

$$S = \frac{S_C e^{-k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)t}}{e^{-k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)t_1}}$$

$$S_F = \frac{S_C e^{-k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)t_2}}{e^{-k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)t_1}}$$

$$\frac{S_F}{S_C} e^{-k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)t_1} = e^{-k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)t_2}$$

$$\ln\left(\frac{S_F}{S_C}\right) - k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)t_1 = -k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)t_2$$

$$t_2 = \frac{\ln\left(\frac{S_F}{S_C}\right) - k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)t_1}{-k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)}$$

$$P = -k_{product} k_{depletion} S\left((S_0 - S_C)Y_S + C_0\right)t$$

$$P(t_2) = -k_{product} k_{depletion} S_F\left((S_0 - S_C)Y_S + C_0\right)t_2$$

$$P(t_2) = \left(k_{product} k_{depletion} S_F\left((S_0 - S_C)Y_S + C_0\right)\right)\left(\frac{\ln\left(\frac{S_F}{S_C}\right) - k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)t_1}{k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)}\right)$$

$$P(t_2) = \left(k_{product} k_{depletion} S_F\left((S_0 - S_C)Y_S + C_0\right)\right)\left(\frac{\ln\left(\frac{S_F}{S_C}\right) - k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)\left(\frac{\ln\left(\frac{(S_0 - S_C)Y_S}{C_0} + 1\right)}{\mu_{\max}}\right)}{k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)}\right)$$

$$P(t_2) = \left(k_{product} k_{depletion} S_F\left((S_0 - S_C)Y_S + C_0\right)\right)\left(\frac{\ln\left(\frac{S_F}{S_C}\right)}{k_{depletion}\left((S_0 - S_C)Y_S + C_0\right)} - \frac{\ln\left(\frac{(S_0 - S_C)Y_S}{C_0} + 1\right)}{\mu_{\max}}\right)$$