14.581: International Economics I
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Pol Antràs

# Problem Set 3

This problem set is due on April 19th in class.

## Exercise 1

Consider a world with two countries, the North and the South, and a unique factor of production, labor. There is a representative consumer in each country with quasi-linear preferences:

$$U = x_0 + \frac{1}{\mu} X^{\mu}, \ 0 < \mu < 1.$$

where $x_0$ is consumption of a homogeneous good, $X$ is an index of aggregate consumption of differentiated varieties, and $\mu$ is a parameter.

Aggregate consumption in the differentiated good sector is a CES function

$$X = \left[ \int x(i)^{\alpha} di \right]^{1/\alpha}, \ 0 < \mu < \alpha < 1,$$

of the consumption of different varieties $x(i)$, where the range of $i$ is endogenous.

Producers of differentiated goods face a perfectly elastic supply of labor. Let the wage in the North be strictly higher than that in the South ($w^N > w^S$). The market structure in that sector is one of monopolistic competition. As in Melitz (2003), producers needs to incur a sunk entry cost equal to $f_E$ units of labor, after which they learn their productivity $\theta$, which is distributed according to a cdf $G(\theta)$. Entry can only occur in North. Final-good production combines two specialized inputs according to the technology:

$$x(i) = \theta \left( \frac{h(i)}{\eta} \right)^{\eta} \left( \frac{m(i)}{1-\eta} \right)^{1-\eta}, \quad 0 < \eta < 1.$$

You can think of $h(i)$ as representing headquarter services and $m(i)$ as representing assembly or basic manufacturing.

Production of the two inputs uses labor under constant returns to scale. One unit of labor produces one unit of $m(i)$ in any country, while one unit of labor in North produces one unit of $h(i)$. South is very inefficient at producing $h(i)$.

Firms that enter in North can produce $h(i)$ in North and $m(i)$ in South, but this entails an additional fixed cost of $f_M$ units of Northern labor.

1. Solve for the demand faced by the unique producer of variety $i$ as a function of $X$ and the price $p(i)$ of this variety. Interpret the condition $\mu < \alpha$.

2. Suppose that $h(i)$ and $m(i)$ are produced in the North. Invert the demand function to express profits as a function of $h(i)$, $m(i)$, $X$, $w^N$, and parameters.

3. Solve for the optimal $h(i)$ and $m(i)$ and express profits as a function of $X$, $w^N$, and parameters. At which price is the final good sold?

4. Suppose now that production of $m(i)$ is offshored to the South. Following similar steps as before, express profits of this strategy as a function of $X$, $w^N$, $w^S$, and parameters.

5. Solve for the threshold productivity level $\theta$ over which offshoring is optimal (continue to treat $X$ as parametric). How does it depend on $w^N$, $w^S$, and $X$?

6. Write down the free entry condition that would allow you to solve for $X$.

   Let us now make the following additional assumptions. First, even when firms do not offshore and have the input be produced in North, they need to incur a fixed cost of production equal to $f < f_M$ units of Northern labor. This fixed cost need not be incurred if the firm decides to exit upon observing $\theta$.

   Assume also that $G(\theta)$ is a Pareto distribution with shape $z$, i.e.,

$$G(\theta) = 1 - \left(\frac{b}{\theta}\right)^z \text{ for } \theta \geq b > 0. \tag{1}$$

7. Solve for the threshold $\theta$ under which firms will decide to exit upon observing $\theta$ (continue to treat $X$ as parametric). How does it depend on $w^N$, $w^S$, and $X$?

8. Solve for the fraction of firms that offshore as a share of firms that do not exit. How does it depend on $w^N$, $w^S$, $X$, and $\eta$? Provide intuition.

## Exercise 2

Consider the same setup as in the previous exercise. Let us however ignore heterogeneity and assume that $\theta = 1$. Also, assume that the only fixed costs of production are the entry ones, i.e., $f = f_M = 0$.

In order to make the choice between producing $m(i)$ in North or South nontrivial, assume now that offshoring is associated with a weaker enforceability of contracts.

In particular, whenever the whole production process is concentrated in North, production is as described in the previous exercise. Instead, when $m(i)$ is located in South, the Northern producer needs to contract with a Southern supplier, but the initial contract can only stipulate a lump-sum transfer between the parties.

After $h(i)$ and $m(i)$ have been produced, the Northern and Southern producers divide the gains from trade according to symmetric Nash bargaining. Assuming zero outside options, they thus each get one-half of sale revenue.

1. Follow the same steps as in Exercise 1 to solve for profits when $h(i)$ and $m(i)$ are produced in the North.

2. Invert the demand function you obtained in Exercise 1, to express revenue when offshoring is chosen as a function of $h(i)$, $m(i)$, $X$, $w^N$, $w^S$ and parameters.

3. Suppose that the Northern and Southern producers choose $h(i)$ and $m(i)$ simultaneously and non-cooperatively to maximize their payoff in the Nash bargaining. Solve for $h(i)$ and $m(i)$ as a function of $X$, $w^N$, $w^S$ and parameters.

4. Express joint profits from offshoring as a function of $X$, $w^N$, $w^S$, and parameters. At which price is the final good sold?

5. Suppose that through the initial transfer, the Northern producer is able to absorb all profits from the relationship. Under which conditions will offshoring be optimal?

6. How is the choice affected by $w^N$, $w^S$, and $\eta$? Provide intuition. Hint: write the ratio of profits from North production to offshoring and study the comparative statics.

7. Now suppose that the Northern producer can vertically integrate the Southern supplier. Suppose that the only effect of integration is to increase the Northern producer's bargaining power from 1/2 to 3/4. How does the ratio of profits from integration to Southern outsourcing depend on $\eta$? Provide intuition.