Massachusetts Institute of Technology

Department of Physics

Course: 8.09 Classical Mechanics
Term: Fall 2006

Quiz 1

October 4, 2006

Instructions

- **Do not start until you are told to do so.**
- **Solve all problems.**
- Put your name on the covers of all notebooks you are using.
- Show all work neatly in the blue book, label the problem you are working on.
- Mark the final answers.
- Books and notes are not to be used. Calculators are unnecessary.

Useful Formulae

Newton and Basic Kinematics:

$$\begin{aligned}
\vec{F} &= \dot{\vec{p}} = m\vec{a} \quad \text{for} \quad v \ll c \\
\vec{v} &= \vec{v}_0 + \int_{t'=0}^{t'=t} dt' \vec{a} \\
\vec{r} &= \vec{r}_0 + \vec{v}_0 t + \int_{t'=0}^{t'=t} dt' \int_{t''=0}^{t''=t'} dt'' \vec{F}(t'')/m
\end{aligned}$$

Lagrangian and Hamiltonian:

$$L(q, \dot{q}) \;=\; T - U; \qquad H(p, q) = T + U = p\dot{q} - L$$

Euler-Lagrange (without and with constraints):

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = 0; \qquad \frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} + \sum_a \lambda_a \frac{\partial g_a}{\partial x} = 0$$

Polar Coordinates:

$$x = r\sin\theta\cos\phi; \quad y = r\sin\theta\sin\phi; \quad z = r\cos\theta$$

Cylindrical Coordinates:

$$x = r\cos\phi; \quad y = r\sin\phi; \quad z = z$$

Possibly useful integrals:

$$\int \frac{dx}{1+x^2} = arctan(x); \quad \int \frac{dx}{1-x^2} = arctanh(x)$$

Problem 1: Variable Length Pendulum (35 points)

Consider a simple pendulum consisting of mass m attached to a string of length L. After the pendulum is set into motion (at $t = 0$), the length of the string is lenghtened at a constant rate

$$\frac{dL}{dt} = \alpha = \text{constant}$$

The suspension point remains fixed.

a) How many degrees of freedom does the system have ?

b) Write the Lagrangian for the pendulum. Calculate $\frac{\partial L}{\partial t}$, is it equal to zero ?

c) Write the equations of motion for the pendulum, do not solve. Show that the equations of motion become the equations of the fixed length pendulum for $\alpha = 0$

d) Write the Hamiltonian for the system.

e) Calculate the total mechanical energy of the system. Compare to the Hamiltonian.

f) The energy of the system is not conserved. What is the rate of change ? Give physical interpretation of the sources and magnitude of power flowing in or out of the system.

Problem 2: Particle on a rotating wire (35 points)

A particle of mass m is constrained to move along a straight frictionless wire. The wire is rotating in a vertical plane at a constant angular velocity Ω as shown in the Figure 1. The system is in the gravitational field with gravitational acceleration g pointing downwards. At $t = 0$ the mass was stationary with respect to the wire and it was at a distance R_0 from the axis of rotation.

a) Write the Lagrangian for a free mass constrained to the vertical plane in gravitational field using polar coordinate system.

b) Write explicitly the constraint equations that force the mass to remain on the rotating wire.

c) Write equations of motion for all the coordinates introducing Lagrange Multipliers.

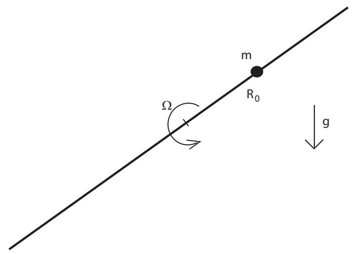

Figure 1: Particle on a rotating wire

d) Obtain the expressions for the forces of constraints. Give the physical interpretation of these forces.

e) For sufficiently large Ω the mass will be moving away from the rotation axis for all wire positions. What is the minimum value of the angular velocity such that this is guaranteed?

Problem 3: Double Pendulum (30 points)

Consider a system consisting of two masses m_1 and m_2 and connected with massless, rigid rods of length L_1 and L_2 (see Figure 2). As the pendulum moves both masses remain in the vertical plane.

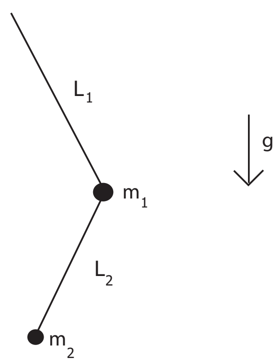

Figure 2: Double Pendulum

a) How many degrees of freedom does the system have?

b) Write the Lagrangian for the system.

c) Write equations of motion for the system. Do not solve.