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18.01 Single Variable Calculus
Fall 2006

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18.01 Solutions to practice questions (Exam 1)

1 a) $D \frac{3t}{\ln t} = \frac{3 \ln t - 3t \cdot \frac{1}{t}}{\ln^2 t} = \frac{3(\ln t - 1)}{\ln^2 t}$

When $t = e^2$
 $\ln t = 2$ due to $\ln t = 2$: $= \frac{3(2-1)}{4} = \frac{3}{4}$

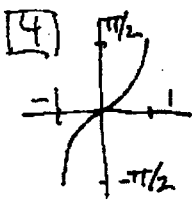
b) $\frac{3u}{\tan 2u} = \frac{3u \cdot \cos 2u}{\sin 2u} = \frac{3u \cos 2u}{2 \sin u \cos u}$
 $= \frac{3}{2} \cdot \frac{u}{\sin u} \cdot \frac{\cos 2u}{\cos u} \rightarrow \frac{3}{2} \cdot 1 \cdot 1$

c) $D \sin kx = k \cos kx$
 $D^2 \dots = k^2 (-\sin kx)$
 $D^3 \sin kx = k^3 (-\cos kx)$

d) $\frac{d}{d\theta} (a + k \sin^2 \theta)^{1/3} = \frac{1}{3} (a + k \sin^2 \theta)^{-2/3} \cdot 2k \sin \theta \cos \theta$
 $= \frac{2}{3} \frac{k \sin \theta \cos \theta}{(a + k \sin^2 \theta)^{2/3}}$

2 $\left. \frac{d}{dx} x^3 \right|_{x_0} = \lim_{\Delta x \rightarrow 0} \frac{(x_0 + \Delta x)^3 - x_0^3}{\Delta x}$
 $= \lim_{\Delta x \rightarrow 0} \frac{x_0^3 + 3x_0^2 \Delta x + 3x_0 \Delta x^2 + \Delta x^3 - x_0^3}{\Delta x}$
 $= 3x_0^2$

3 On the one hand, $\left. \frac{d}{dx} \sqrt[3]{x} \right|_{x=1} = \frac{1}{3} x^{-2/3} \Big|_{x=1} = \frac{1}{3}$
 And: by def'n
 $\left. \frac{d}{dx} \sqrt[3]{x} \right| = \lim_{\Delta x \rightarrow 0} \frac{\sqrt[3]{1+\Delta x} - \sqrt[3]{1}}{\Delta x} = \lim_{h \rightarrow 0} \frac{\sqrt[3]{1+h} - 1}{h}$
 $= 1/3$ therefore

4  $y = \sin^{-1} x \Leftrightarrow \sin y = x$
 implicit diff'n:
 $\cos y \cdot y' = 1$
 $y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1 - \sin^2 y}} = \frac{1}{\sqrt{1 - x^2}}$

(since $\cos y > 0$ for $-\pi/2 \leq y < \pi/2$, choose the positive square root.)

5 $f(x) = \begin{cases} ax+b, & x > 0 \\ 1-x+x^2, & x \leq 0 \end{cases}$

Continuous $\Leftrightarrow ax+b \Big|_0 = 1-x+x^2 \Big|_0$, or $b=1$

Diff. \Leftrightarrow continuous and slopes are $=$ at 0:

$a = -1 + 2x \Big|_0$ or $a = -1$

6 $x^2 y + y^3 + x^2 = 8$ \otimes

By implicit diff'n:

$2xy + x^2 y' + 3y^2 y' + 2x = 0$

Slope horizontal $\Leftrightarrow y' = 0$

$\Leftrightarrow 2x(y+1) = 0$

$\Leftrightarrow x=0$ or $y=-1$

$x=0 \Rightarrow y^3 = 8 \Rightarrow y=2$ (0,2)


$y=-1 \Rightarrow -x^2 - 1 + x^2 = 8 \Rightarrow -1 = 8$ impossible
 \therefore at (0,2)

7 tan line at (x_0, y_0) :
 $y - y_0 = f'(x_0)(x - x_0)$ \forall x-intercept, $y=0$
 $\therefore -y_0 = f'(x_0)(x - x_0)$
 $\therefore x = x_0 - \frac{y_0}{f'(x_0)}$, if $f'(x_0) \neq 0$

8 $V = \frac{4}{3} \pi r^3$ vol of sphere
 $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$, by chain rule
 $-10 = 4\pi (20)^2 \frac{dr}{dt}$
 $\frac{dr}{dt} = \frac{-10}{4\pi(20)^2} = -\frac{1}{160\pi}$ cm/sec

9 a) $\sec x = \frac{1}{\cos x}$ $\cos x = 0$ at $x = \pm \pi/2, \pm 3\pi/2, \dots$
 i.e., at $x = \frac{(2n+1)\pi}{2}$, $n=0, \pm 1, \dots$

b) $\frac{1+x^2}{1-x^2} = \frac{1+x^2}{(1-x)(1+x)}$ at $x=1$ $x=-1$

c)  no slope at $x=0$

10 a) $A = A_0 e^{-rt}$
 $\frac{A}{A_0} = \frac{1}{4} = e^{-rt}$ take logs:
 $-\ln 4 = -rt$
 $\therefore t = \frac{\ln 4}{r}$

b) $\frac{dA}{dt} = A_0 e^{-rt} \cdot (-r)$
 $= A \cdot (-r)$
 $= \frac{1}{4} (-r) = -\frac{r}{4}$