

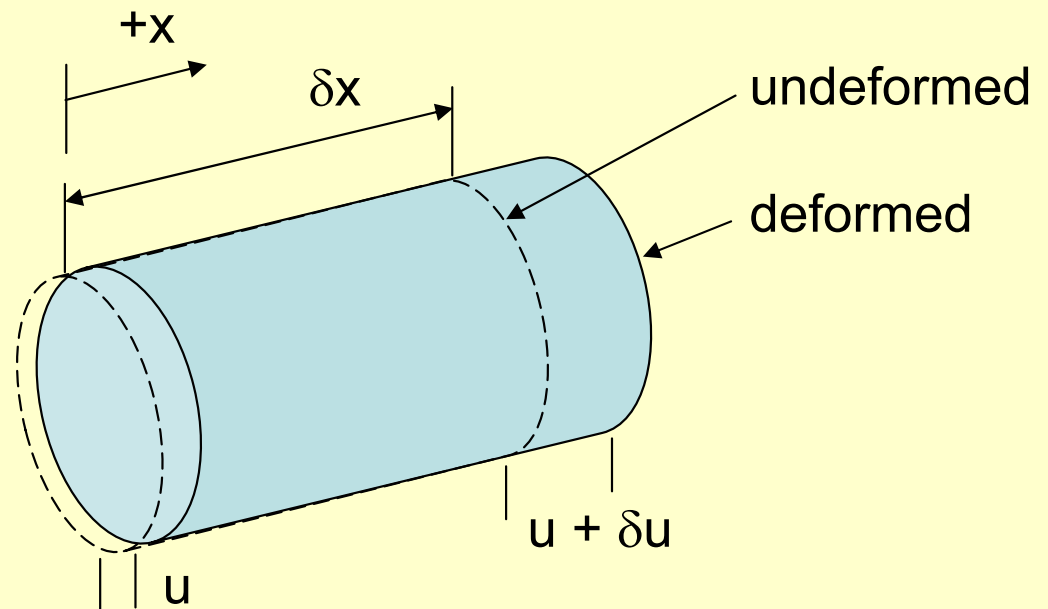
# Basic Physics of Underwater Acoustics

*Reference used in this lecture: Lurton, X. 2002. An introduction to underwater acoustics. New York: Springer.*

# Definitions

- $p$ : pressure, measured *relative to hydrostatic*, Pa  
 $\rho$ : density, measured *relative to hydrostatic*, kg/m<sup>3</sup>  
 $E$ : bulk modulus of the fluid, Pa,  $\delta p = E [ \delta \rho / \rho ]$   
 $[u,v,w]$ : deflections in  $[x,y,z]$ -directions, relative to the hydrostatic condition, m

Then in one dimension (*pipe*)  
 $p = E [ -\delta u / \delta x ]$



# One-dimensional Case *cont.*

Newton's Law:

$$\delta p = -\rho u_{tt} \delta x \quad \text{OR}$$

$$p_x = -\rho u_{tt}$$

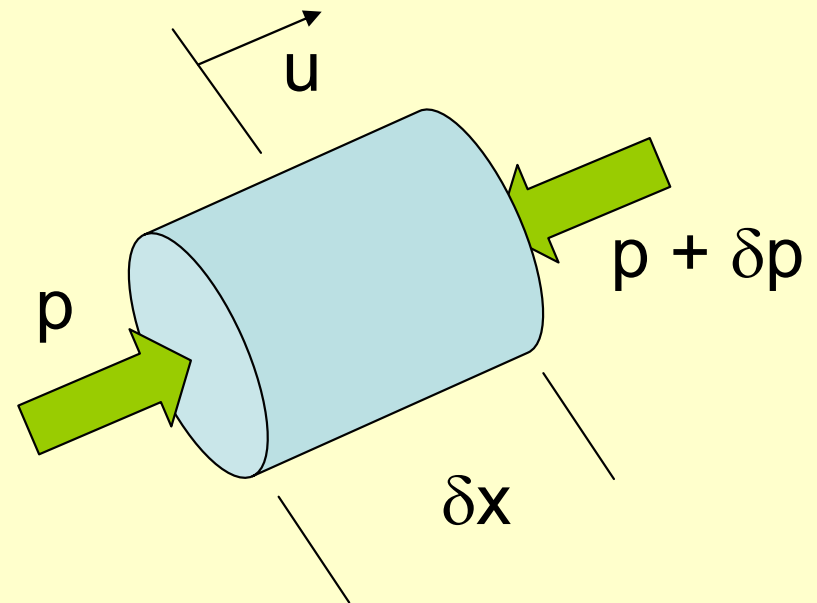
diff wrt  $x$

Constitutive Law:

$$p = -E \delta u / \delta x \quad \text{OR}$$

$$p = -E u_x$$

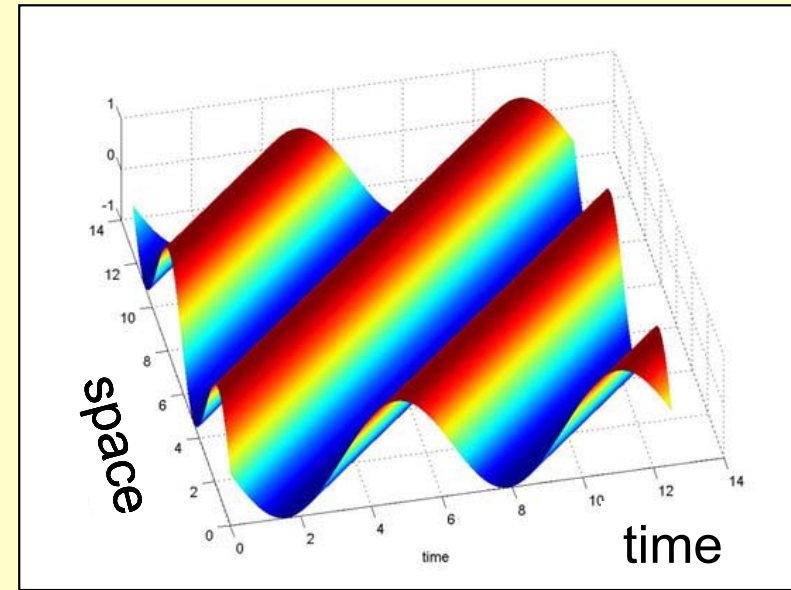
diff wrt  $tt$



$$p_{xx} = [\rho / E] p_{tt}$$

a wave equation!

Let  $p(x,t) = P_o \sin(\omega t - kx) \longrightarrow$



Insert this in the wave equation:

$$- P_o k^2 \sin( ) = - [ \rho / E ] P_o \omega^2 \sin( ) \rightarrow$$
$$[ \omega / k ]^2 = E / \rho \rightarrow$$

Wave speed  $c = \omega / k = [ E / \rho ]^{1/2}$

This is *sound speed in water*, independent of pressure, or frequency.

$\rho \sim 1000 \text{ kg/m}^3$ ,  $E \sim 2.3\text{e}9 \text{ N/m}^2 \rightarrow c \sim 1500 \text{ m/s}$

Wavelength  $\lambda = 2\pi/k = 2\pi c/\omega = c/f$ ; **1kHz : 1.5m**

# In Three Dimensions: A CUBE

Newton's Law:

$$p_x = -\rho u_{tt} \rightarrow p_{xx} = -\rho u_{ttx}$$

$$p_y = -\rho v_{tt} \rightarrow p_{yy} = -\rho v_{tty}$$

$$p_z = -\rho w_{tt} \rightarrow p_{zz} = -\rho w_{ttz}$$

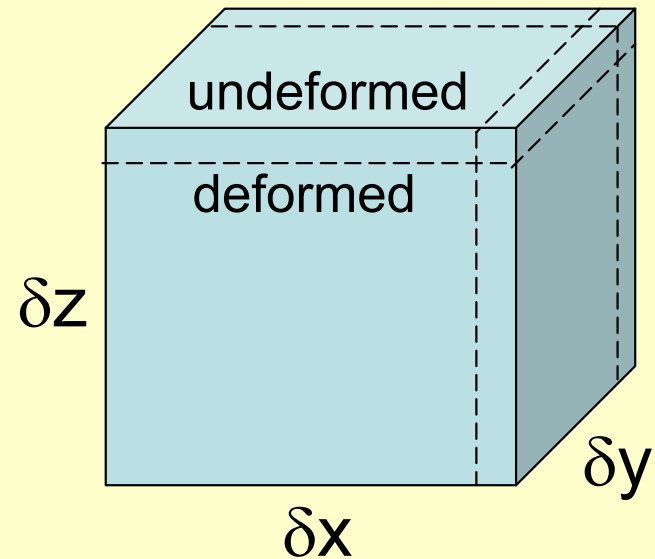
Constitutive Law:

$$-E u_x = p / 3 \rightarrow -E u_{ttx} = p_{tt} / 3$$

$$-E v_y = p / 3 \rightarrow -E v_{tty} = p_{tt} / 3$$

$$-E w_z = p / 3 \rightarrow -E w_{ttz} = p_{tt} / 3$$

*All directions deform uniformly*



*Lead to Helmholtz Equation:*

$$p_{xx} + p_{yy} + p_{zz} = p_{tt} / c^2$$

$$\text{or } \nabla^2 p = p_{tt} / c^2$$

*where  $\nabla^2$  is the Laplacian operator*

# Particle Velocity

Consider one dimension again (Newton's Law):

$$p_x = -\rho u_{tt} \rightarrow p_x = -\rho (u_t)_t$$

$$\text{If } p(x,t) = P_o \sin(\omega t - kx) \text{ and } u_t(x,t) = U_{to} \sin(\omega t - kx) \rightarrow$$

$$-kP_o \cos(\ ) = -\rho \omega U_{to} \cos(\ ) \rightarrow U_{to} = P_o / \rho c$$

*Note velocity is in phase with pressure!*

$[\rho c]$ : characteristic impedance;

water:  $\rho c \sim 1.5e6$  Rayleighs “hard”

air:  $\rho c \sim 500$  Rayleighs “soft”

In three dimensions:

$$\nabla p = -\rho \underline{V}_t \text{ where}$$

$$\nabla p = p_x i + p_y j + p_z k \text{ and}$$

$$\underline{V} = u_t i + v_t j + w_t k$$

Note equivalence of the following:

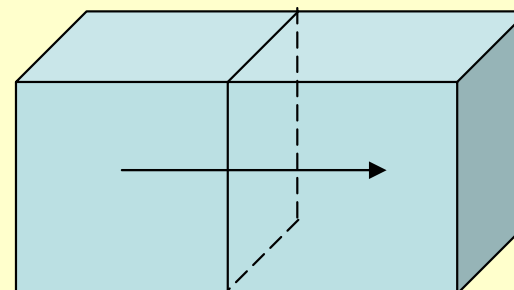
$$\lambda = c / f \quad \text{and} \quad \omega / k = c$$

There is no dispersion relation here; this is the only relationship between  $\omega$  and  $k$ !

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Consider Average Power through a 1D surface:

$$\begin{aligned} \mathbf{P}(x) &= [ 1 / T ] \int^T p(\tau, x) u_t(\tau, x) d\tau \\ &= [ 1 / T ] \int^T P_o U_{to} \sin^2(\omega\tau - kx) d\tau \\ &= P_o U_{to} / 2 \\ &= P_o^2 / 2 \rho c = U_{to}^2 \rho c / 2 \\ &\quad \underline{\text{Acoustic Intensity in W/m}^2} \end{aligned}$$



*Power per unit area is pressure times velocity*

If impedance  $\rho c$  is high, then it takes little power to create a given pressure level; but it takes a lot of power to create a given velocity level

# Spreading in Three-Space

At time  $t_1$ , perturbation is at radius  $r_1$ ; at time  $t_2$ , radius  $r_2 \rightarrow$

$$\mathbf{P}(r_1) = P_o^2(r_1) / 2 \rho c$$

$$\mathbf{P}(r_2) = P_o^2(r_2) / 2 \rho c$$

Assuming no losses in water; then

$$\mathbf{P}(r_2) = \mathbf{P}(r_1) r_1^2 / r_2^2 = P_o^2 (r_1) r_1^2 / 2 \rho c r_2^2$$

and

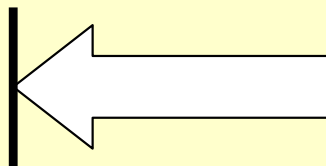
$$P_o(r_2) = P_o(r_1) r_1 / r_2$$

Let  $r_1 = 1$  meter (standard!)  $\rightarrow$

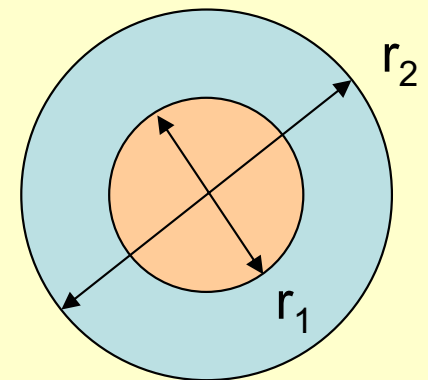
$$\mathbf{P}(r) = P_o^2(1m) / 2 \rho c r^2$$

$$P_o(r) = P_o(1m) / r$$

$$U_{to}(r) = P_o(1m) / \rho c r$$



*Pressure level and particle velocity decrease linearly with range*



# Decibels (dB)

$10 * \log_{10}$  (ratio of two positive scalars):

Example:

$x_1 = 31.6$  ;  $x_2 = 1 \rightarrow$  1.5 orders of magnitude difference

$$10 * \log_{10}(x_1/x_2) = \mathbf{15dB}$$

$$10 * \log_{10}(x_2/x_1) = \mathbf{-15dB}$$

RECALL  $\log(x_1^2/x_2^2) = \log(x_1/x_2) + \log(x_1/x_2) = 2 \log(x_1/x_2)$

In acoustics, *acoustic intensity (power)* is referenced to **1 W/m<sup>2</sup>** ;  
*pressure* is referenced to **1 μPa**

$$\begin{aligned} 10 * \log_{10} [ \mathbf{P}(r) / 1 \text{ W/m}^2 ] &= 10 * \log_{10} [ [ P_o^2(r) / 2 \rho c ] / 1 \text{ W/m}^2 ] \\ &= 20 * \log_{10} [ P_o(r) ] - 10 * \log_{10}(2\rho c) \\ &= 20 * \log_{10} [ P_o(r) / 1\mu\text{Pa} ] - 120 - 65 \end{aligned}$$

# Spreading Losses with Range

Pressure level in dB at range  $r$  is

$$20 \log_{10} [ P_o(r) / 1\mu\text{Pa} ] - 185 =$$

$$20 \log_{10} [ P_o(1\text{m}) / r / 1\mu\text{Pa} ] - 185 =$$

$$20 \log_{10} [ P_o(1\text{m}) / 1\mu\text{Pa} ] - \underline{20 \log_{10} [r]} - 185$$

**Example:** At 100m range, we have lost  
40dB or *four orders of magnitude* in sound intensity  
40dB or *two orders of magnitude* in pressure  
(and particle velocity)

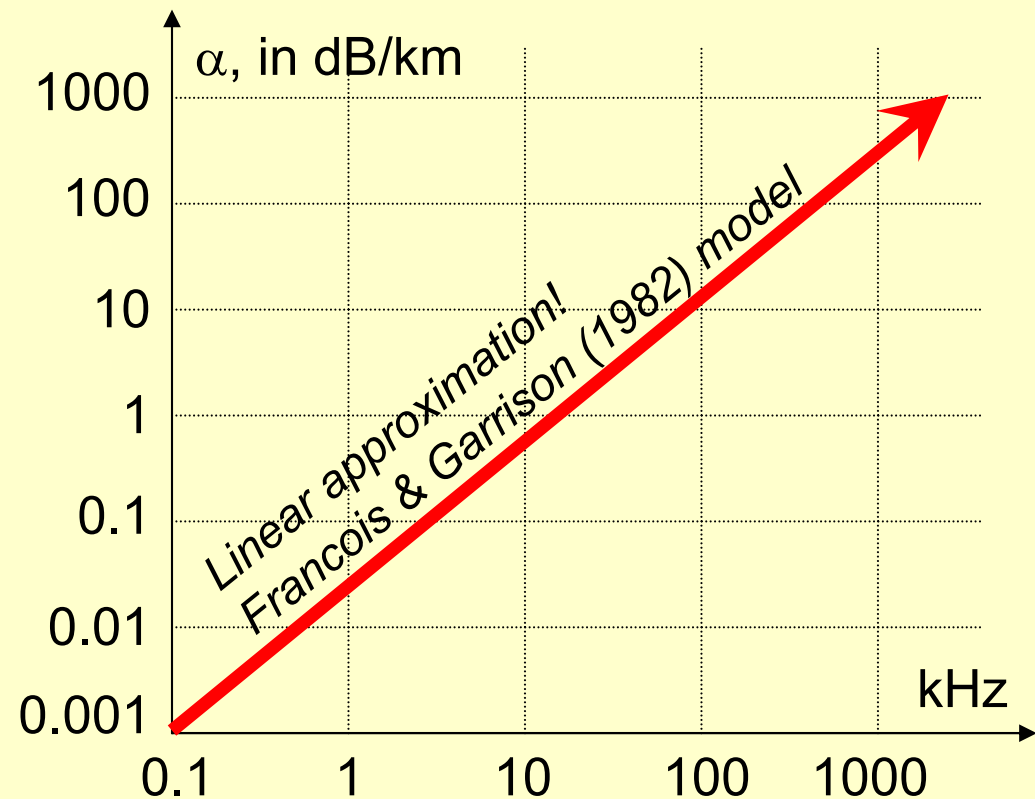
# Attenuation Losses with Range

Acoustic power does have losses with transmission distance – primarily related to relaxation of boric acid and magnesium sulfate molecules in seawater. Also bubbles, etc.

At 100 Hz, ~1dB/1000km:  
OK for thousands of km,  
ocean-scale seismics and  
communications

At 10kHz, ~1dB/km:  
OK for ~1-10km,  
long-baseline acoustics

At 1MHz, 3dB/10m:  
OK for ~10-100m,  
imaging sonars, Doppler  
velocity loggers



$$TL = 20 \log_{10} r + \alpha r$$

(pressure  
transmission loss)

# The Piezo-Electric Actuator

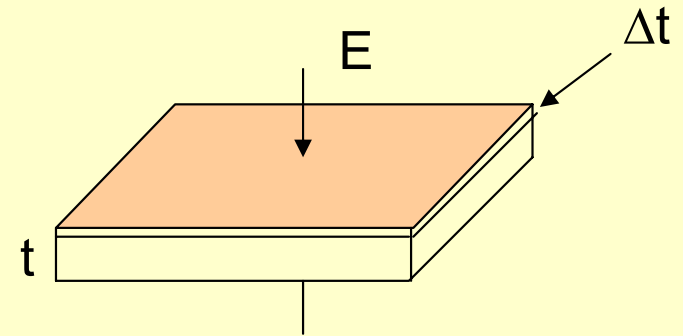
strain = constant  $\times$  electric field

$$\varepsilon = d \times E \quad \text{or}$$

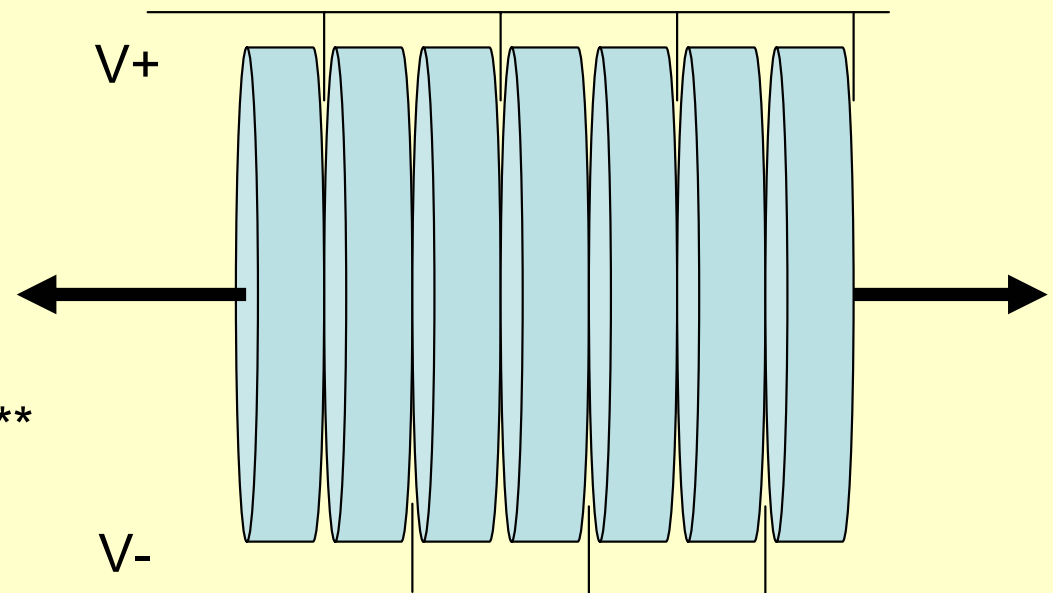
$$\Delta t / t = d \times (V / t)$$

where  $d = 40\text{-}750 \times 10^{-12} \text{ m / V} \rightarrow$

Drive at 100V, we get only 4-75 nm thickness change!



Series connection  
amplifies  
displacement



\*\*still capable of MHz performance\*\*

# The Piezo-Electric Sensor

electric field = constant X stress

$$E = g X \sigma \quad \text{or}$$

$$V = t g \sigma$$

where  $g = 15\text{-}30 \times 10^{-3} \text{ V/mN}$

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Ideal Actuator: Assume the water does not impede the driven motion of the material

Ideal Sensor: Assume the sensor does not deform in response to the water pressure waves

## Typical Transducer:

*120 to 150 dB re 1  $\mu$ Pa, 1m, 1V*

means

$10^6 - 10^{7.5}$   $\mu$ Pa at 1m for each Volt applied

or

1-30 Pa at 1m for each Volt applied

## Typical Hydrophone:

*-220 to -190 dB re 1  $\mu$ Pa, 1V*

means

$10^{-11}$  to  $10^{-9.5}$  V for each  $\mu$ Pa incident

or

$10^{-5}$  to  $10^{-3.5}$  V for each Pa incident

So considering a transducer with 16Pa at 1m per Volt, and a hydrophone with  $10^{-4}$  V per Pa:

If  $V = 200V$ , we generate 3200Pa at 1m, or 3.2Pa at 1km, assuming spreading losses only;

The hydrophone signal at this pressure level will be 0.00032V or 320 $\mu$ V !