Problem Set 2 Answer Key

1) Good job on this part. Everyone did well.

Basic requirements for this question:

0.5 dpm = 262800 decays/y

$t_{1/2}$ (^{210}Pb) = 22.3 y

λ (^{210}Pb) = 0.0311 y^{-1}

$t_{1/2}$ (^{231}Pa) = 32760 y

λ (^{231} Pa) = 2.166 x 10^{-5} y^{-1}

a) For counting techniques:

N_{210} = (262800 decays/y)/(0.0311 y^{-1}) = 8.45 x 10^6 atoms < 10^8

N_{231} = (262800 decays/y)/(2.116 x 10^{-5-} y^{-1}) = 1.242 x 10^{10} atoms > 10^8

Use counting techniques for ^{210}Pb and mass spectrometry for ^{231}Pa.

b) For ^{210}Pb:

10^8 – 8.45 x 10^6 = 9.16 x 10^7 atoms

or

(8.45 x 10^6)/10^8 = 8 %

For ^{231}Pa:

1.242 x 10^{10} – 10^8 = 1.232 x 10^{10} atoms

or

10^8/(1.242 x 10^{10}) = 0.8 %

c) Mass spec. becomes advantageous when the activity is less then the number of atoms times the decay constant.

$A < N*\ln(2)/t_{1/2}$

A = 0.5 atoms/minute = 2.628 x 10^5 atoms/y

N = 10^8 atoms

$t_{1/2}$ (minimum) = N*ln(2)/A = (10^8 atoms)*ln(2)/(2.628 x 10^5 atoms/y) = 264 years

2) This question has some subtleties that not everyone picked up on. Everyone did very well using the basic equations. The problems stemmed from two parts: the assumption of secular equilibrium and the use of relative abundances.

Isotope	$t_{1/2}$ (y)	λ (y^{-1})	Ratio vs. ^{238}U at t = 0
^{238}U	4.468 x 10^9	1.551 x 10^{-10}	1
^{235}U	7.038 x 10^8	9.849 x 10^{-10}	0.00725
^{234}U	2.455 x 10^5	2.823 x 10^{-6}	0.0000554

The relative abundances that are given in this question cannot just be plugged in to the decay equation. The decay equation calls for N, which is in atoms. However, if we create a hypothetical situation that we assume that there are 100 atoms of U today. So there would be 99.2745 atoms of ^{238}U, 0.7196 atoms of ^{235}U and 0.0055 atoms of ^{234}U. From this we can calculate the number of atoms of each at t = 0. However, this is not the 'ideal' way to do this, because we make some assumptions that we don't have to make (however, this was an acceptable answer). So, here we go on the preferable method:

^{238}U and ^{235}U are the start of two decays series. Therefore, they were produced during nucleosynthesis, and have no other source (well, other than us… but we will ignore human additions for now). So to calculate the amounts of these at t = 0 we can use the equation:

$$N = N_o e^{-\lambda t}$$

However, we don't have the number of atoms, but we do have their relative abundances, and through that we have the abundances relative *to each other*.

$$\frac{^{238}N}{^{235}N} = \frac{^{238}N_o e^{-\lambda 238 * t}}{^{235}N_o e^{-\lambda 235 * t}}$$

$$^{238}N/^{235}N = R$$

$$R_o = Re^{[-(\lambda 238 - \lambda 235)*t]}$$

$$R_o = (0.00725)*e^{[-(9.849E-10 - 1.551E-10)*4.6E9]}$$

$$R_o = 0.323$$

$$^{235}N_o = 0.323 * {}^{238}N_o$$

The next step is to make an assumption about ^{234}U. ^{234}U is produced during nucleosynthesis and in the decay series of ^{238}U. However, we can simplify the problem by assuming that *at t = 0 ^{234}U is in secular equilibrium with ^{238}U.* Now is this a good assumption? Well, that issue is addressed by considering the half-life of ^{234}U and the length of time between nucelosynthesis and the formation of the earth. It turns out that ^{234}U's half-life is short compared to the time of earth formation, so our assumption is good. With this assumption made we can state:

$$^{234}N_o \lambda_{234} = {}^{238}N_o \lambda_{238}$$

$$^{234}N_o = (\lambda_{238}/\lambda_{234})^{238}N_o$$

$$^{234}N_o = (1.551 \text{ x } 10^{-10}/2.823 \text{ x } 10^{-6}) * {}^{238}N_o$$

$$^{234}N_o = (5.494 \text{ x } 10^{-5}) * {}^{238}N_o$$

Now we know the relative amounts of each isotope at t = 0, relative to ^{238}U. And we know that the three isotopes add up to the total amount of U (100%). Since we are looking for the relative abundances of each isotope at t = 0, we solve as follows:

$$^{238}U + {}^{235}U + {}^{234}U = U_T$$

$$^{238}N + 0.323(^{238}N) + 5.494 \text{ x } 10^{-5}(^{238}N) = N_T$$

$$^{238}N/N_T + 0.323(^{238}N/N_T) + 5.494 \text{ x } 10^{-5}(^{238}N/N_T) = 1$$

$$1.323(^{238}N/N_T) = 1$$

$$^{238}U = 1/1.323 = 75.6\%$$

$$^{235}U = 0.323(75.6\%) = 24.4\%$$
$$^{234}U = 5.494 \times 10^{-5}(75.6\%) = 0.415\%$$

3. Every did a pretty good job with this question, too. Some small problems arose on the last part, but we will get to that later. To start, the basis behind all the parts to this problem is obviously in = out. Similar to the assumptions we make with steady state, in this case we make the assumption that the earth maintains a radiative balance.

a) No atmosphere. So in this case we have a very simple model. The energy that comes into the earth (F_{in}) with is reflected back to space (αF_{in}) or is absorbed by the earth and re-emitted (S).

$$F_{in} = F_{out}$$
$$F_{in} = S_o/4 = 342 \text{ W/m}^2$$
$$F_{out} = \alpha F_{in} + S$$
$$F_{in} = \alpha F_{in} + S$$
$$S = (1-\alpha)F_{in}$$
$$S = 0.67*342 \text{ W/m}^2$$
$$S = 229.14 \text{ W/m}^2$$

$$T_S = (S/\sigma)^{1/4}$$
$$T_S = [(229.14 \text{ W/m}^2)/(5.7E\text{-}8 \text{ W/m}^2K^4)]^{1/4}$$
$$T_S = 252 \text{ K}$$

b) Now it gets a bit more complicated, we need to assume a homogeneous atmosphere that allows the incoming solar radiation to pass through it, but absorbs all of the outgoing planetary radiation. The sketch should make it clear that the atmosphere is also transparent to the reflected solar radiation – remember, it has the same wavelength as the incoming solar radiation. It is not until the energy is absorbed and re-radiated that the energy can be absorbed by the atmosphere.

Energy balance at the surface: $F_{in} + A = \alpha F_{in} + S$
Energy balance of the atmosphere: $S = 2A$

$$F_{in} + A = \alpha F_{in} + 2A$$
$$A = F_{in}(1-\alpha)$$

$$T_A = [(342 \text{ W/m}^2 * 0.67)/(5.7E\text{-}8 \text{ W/m}^2K^4)]^{1/4} = 252 \text{ K}$$
$$T_S = (2*4.02E+9 \text{ K}^4)^{1/4} = 299 \text{ K}$$

c) Now things get even more complicated – the atmosphere no longer completely absorbs the planetary radiation, but only a portion of it (ε).

Energy balance at the surface: $F_{in} + A = \alpha F_{in} + S$
$S = (1-\alpha)F_{in} + A$

Energy balance of the atmosphere: $S = 2A + (1-\varepsilon)S$

$$S = 2A/\varepsilon$$

$$S = S$$
$$(1-\alpha)F_{in} + A = 2A/\varepsilon$$
$$A = (1-\alpha)F_{in}/((2/\varepsilon) - 1)$$
$$A = 169.4 \text{ W/m}^2$$
$$S = 398.6 \text{ W/m}^2$$

$$T_A = (A/\sigma\varepsilon)^{1/4} = 243 \text{ K}$$
$$T_S = (S/\sigma)^{1/4} = 289 \text{ K}$$

d) The discrepancy between the observed temperature of the earth's surface and the calculated temperature (above) is due to the assumptions and simplifications that we made in order to answer the question. The main simplification is that the atmosphere is a single homogenous box. It is, of course, far more complicated, and this can have a large effect on the temperature of the earth's surface.

4. I tried to comment on each of your problem sets. If there are any additional questions do not hesitate to contact me. Here are some general answers:

(10) a) Winds tend to flow along lines of constant pressure (isobars), and flow the most quickly when the isobars are closely set. The air wants to move from areas of high pressure to areas of low pressure, but the rotation of the earth causes the Coriolis effect and the winds are deflected to the right (left) in the Northern (Southern) hemisphere.

(10) b) The main point here was that the two maps are not sampled simultaneously, nor are they updated with the same frequency (5). This inevitably leads to discrepancies between the two maps, though the large-scale features are generally accurate. Also, the NCEP map is a analysis map (a compilation of data that is analyzed by the NOAA scientists to give that maps posted on the website), while the QuikSCAT map is an observation. They serve different purposes and contain different information (5).

(5) c) The climatological fields that were discussed in class are the long-term averaged wind speeds and directions. The QuikSCAT map is instantaneous and discrete. Therefore variations that are small in time and space, but large in magnitude may not show up on a climatological field. This is important in gas exchange (as you will soon see) because it is dependant upon wind speed in a non-linear fashion. For example, storms are not included in climatological fields and they have very large effects on the amount of gas exchange that occurs over a finite time period. So an estimate of gas exchange based on climatological fields could be drastically inaccurate.