14.581: International Economics I
Spring 2007
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Problem Set 4

This problem set is due on May 15th in class.

Exercise 1

Consider a collection of M economies each producing a single good using capital and labor under a country-specific neoclassical production function

$$Y_j(t) = F(K_j(t), A_j(t) L_j(t)), \quad j = 1, ..., M,$$

featuring constant returns to scale, diminishing marginal products and the standard Inada conditions. Suppose that, as in the Solow model, consumers save a constant fraction s_j of output in any given period, and that this saved output is transformed one to one into capital. The savings rate is country specific. Similarly, population grows at a constant rate n_j in country $j = 1, ..., M$. Capital depreciates at rate δ everywhere.

Countries are not allowed to trade in goods or assets, but there is a process of international technology diffusion. In particular, $A_j(t)$ evolves over time according to

$$\dot{A}_j(t) = \sigma_j(A(t) - A_j(t)) + \lambda_j A_j(t),$$

where $A(t)$ is the "world technology frontier" and satisfies $A(t) \geq A_j(t)$ for all j and t. Let this world frontier grow at a rate g, that is, $\dot{A}(t)/A(t) \equiv g$.

1. Suppose first that $\sigma_j = 0$. Write down the law of motion for capital and solve for the balanced-path growth rate in country $j = 1, ..., M$. What is the steady-state value of the ratio $k_j(t) \equiv K_j(t)/(A_j(t) L_j(t))$?

2. Consider next the case in which $\sigma_j > 0$. Interpret the parameter σ_j. Express the law of motion of $A_j(t)$ in terms of $a_j(t) \equiv A_j(t)/A(t)$. What is the steady-state value of $a_j(t)$? **Extra points:** Prove formally that the steady state is unique

3. Solve for the balanced-path growth rate in country $j = 1, ..., M$. How does it depend on λ_j?

4. Solve for the steady-state value of the ratio $k_j(t) \equiv K_j(t) / (A_j(t) L_j(t))$.
5. Solve for the steady-state level of income per capita. How does it depend on λ_j, σ_j, and s_j? Interpret.

Exercise 2

Consider a world with two countries (H and F) that produce two goods (food and manufactures) using a unique factor of production (labor). Both countries are endowed with an equal amount of labor L.

Food is produced with a constant returns to scale technology featuring a unit coefficient in both countries, that is $Y_F^j = L_F^j$, where L_F^j is the amount of labor that country j employs in food production.

Manufactures are also produced linearly with labor, but they consist of a series of increasingly sophisticated generations of goods. These generations are assumed to be perfect substitutes. Country j's productivity in generation i at time t is country-specific and is a function of past production of that generation of manufactures in that country. Hence, this model features learning-by-doing that is country- and sector/generation- specific. In particular, assume

$$Y_i^j(t) = A_i\left(K_i^j(t)\right) L_i^j(t),$$

where L_i^j is the amount of labor that country j allocates to producing generation i of manufactures, and

$$K_i^j(t) = \int_{-\infty}^{t} Y_i^j(\tau)\, d\tau.$$

We will sometimes denote $A_i\left(K_i^j(t)\right)$ simply by A_i^j (with the understanding that A_i^j varies through time). Assume that the function $A_i(\cdot)$ is increasing and strictly concave. Assume also that for a given accumulated amount of output Z, we have

$$A_{i+i}(Z) > A_i(Z),$$

so larger index generations are, ceteris paribus, more productive.

On the demand side, consumers have Cobb-Douglas preferences and allocate a fraction $\beta > 1/2$ of their spending to manufactures.

1. Consider first the initial static equilibrium in which $A_1^H > A_1^F$ (because of larger production at Home in the past) and $A_i^j = 0$ for all $i > 1$. Show that there are only

two types of equilibria: one with complete specialization and one with both countries producing manufactures of generation 1.

2. Solve for relative wages and the allocation of factors to each sector in each of these types of equilibria. Solve for the real wage (nominal wage divided by the ideal price index) in each country.

3. Assume that $A_1^H/A_1^F > \beta/(1-\beta)$. Describe the law of motion for A_1^H and A_1^F.

4. Now assume that at some date T_1 a "second-generation" technology in manufacturing becomes available. Assume that

$$A_2(0) < A_1\left(K_1^H(T_1)\right).$$

What does this condition ensure?

5. Assume also that

$$A_2(0) > \left(\frac{1-\beta}{\beta}\right) \times A_1\left(K_1^H(T_1)\right).$$

What does this condition ensure? How is the pattern of specialization affected? Solve for the new relative wages in North and South.

6. In the periods immediate following T_1, what will happen to the ratio A_1^H/A_2^F if the function $A_2(\cdot)$ is steep enough around zero? What will happen to relative wages w^H/w^F and real wages in each country?

7. Suppose that the function $A_1(\cdot)$ and $A_2(\cdot)$ are such that at some point $T_2 > T_1$, we have $A_2^F/A_1^H > \beta/(1-\beta)$. What will happen after T_2? Provide intuition for the result.

Exercise 3

Consider a world consisting of two countries: Home and Foreign. Each country is populated by a continuum of measure 1 of individuals with identical preferences:

$$u^j = c_0^j + \sum_{i=1}^{2} u_i^j\left(c_i^j\right), \quad j = H, F$$

where $u_i^j(\cdot)$ is increasing and strictly concave. Good 0 serves as the numeraire, is costlessly traded and not subject to tariffs. Its world and domestic price is normalized to 1. It is produced one to one with labor everywhere in the world, which pins down the wage rate to 1 in all countries. The other goods can also be traded internationally, but for one unit of good

i to make it to the other country, $d_i > 1$ units have to be shipped. Non-numeraire goods are produced combining labor and sector-specific capital according to a constant returns to scale technology. Let $\Pi_i^j \left(p_i^j\right)$ be the rent accruing to sector i specific factor in country j.

For simplicity, assume that good 1 is a "natural export" of Home, while good 2 is a "natural export" of Foreign, in the sense that trade policy cannot revert "natural" comparative advantage patterns.

Let us focus on a world in which the only tariff policy available to countries is an add-valorem tax on imports. Any tariff revenue is rebated back to consumers in a lump-sum manner. Let p_i^W denote the world untaxed price of good i. This corresponds to the price paid by consumers in the exporting country.

Suppose first that the government in each country j sets the tariff level that maximizes welfare in the country.

1. Follow the steps in Lecture 20 and solve for the optimal import tariff in each country as a function of the export supply elasticities.

2. How does the optimal tariff depend on transportation costs? Provide intuition.

   Assume now that each country's government is partially altruistic and maximizes a weighted sum of domestic and foreign welfare with a weight $\mu < 1$ on welfare abroad.

3. Solve for the optimal tariff in that case. What is the effect of μ on the optimal tariff? What happens when $\mu \to 1$? Provide intuition.

   Impose now the following functional forms. Utility functions u^j are quadratic, so demand functions are linear:

$$\begin{aligned} c_i^H \left(p_i^H\right) &= \lambda \left(\alpha_i^H - \beta p_i^H\right), \\ c_i^F \left(p_1^F\right) &= \alpha_i^F - \beta p_i^F, \end{aligned}$$

   where $\alpha_2^H = \alpha_1^F = \alpha_L > \alpha_S = \alpha_1^H = \alpha_2^F$. Supply functions are also linear (the rent functions Π_i^j are quadratic):

$$\begin{aligned} y_i^H \left(p_i^H\right) &= \lambda \left(a + b p_i^H\right) \\ y_i^F \left(p_i^F\right) &= a + b p_i^F \end{aligned}$$

4. Solve for the optimal tariff with no altruism. How does it depend on λ? Provide intuition. How does the tariff depend on d_i?

5. Solve for the optimal tariff with altruism. How does it depend on μ and λ?