22.01 Problem Set 3 Homework Solutions

1. (3 points) The activity of a radioisotope is found to decrease to 45% of its original value in 30 days.

(a) *What is the decay constant?*

We are solving for the decay constant, λ. We know that the activity after t=30 days is now .45A_0 of the original activity, A_0. Solving for λ:

$$A = A_0 e^{-\lambda t}$$
$$\frac{A}{A_0} = .45 = e^{-\lambda(30)}$$
$$\therefore \lambda = 0.0266 days^{-1} = 38.3 \min^{-1}$$

(b) *What is the half-life?*

By definition of half-life:

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{.0266} = 26.06 days$$

(c) *What is the mean life?*

By definition of mean life:

$$\tau = \frac{1}{\lambda} = \frac{1}{.0266} = 37.6 days$$

2. (4 points) How long will it take for each of the following radioisotopes to decrease to 0.0001% of its initial activity?

We want to solve for time using the equation:

$$\frac{A}{A_0} = e^{-\lambda t}$$

where A/A_0 is 0.000001 and λ is specific to each of the atoms.

(a) ^{64}*Copper*, $t_{½}$ = 12.7 hr, so λ = 0.055 hr^{-1}, ∴ t = 251.2 hr

(b) ^{41}*Scandium*, $t_{½}$ = 596.3 ms, so λ = 0.0012 ms^{-1}, ∴ t = 1.19x10^4 ms

(c) ^{99}*Technetium*, $t_{½}$ = 2.12x10^5 y, so λ = 3.27x10^{-6} y, ∴ t = 4.23x10^6 y

(d) ^{99m}*Technetium*, $t_{½}$ = 6.02 h, so λ = 0.115 h, ∴ t = 120 h

3. (4 points) A sample contains 1.0 GBq of ^{90}Sr and 0.62 GBq of ^{90}Y.

The decay equation for ^{90}Sr:

$$^{90}_{38}Sr \xrightarrow{t_{1/2}=29.12y} {}^{90}_{39}Y \xrightarrow{t_{1/2}=64h} {}^{90}_{40}Zr(stable)$$

$\lambda_{Sr} = 7.55\text{x}10^{-10}\ s^{-1}$ and $\lambda_Y = 3.008\text{x}10^{-6}\ s^{-1}$

$A_{0,Sr} = 1.0\text{x}10^9\ s^{-1}$ and $A_{0,Y} = 0.62\text{x}10^9\ s^{-1}$

^{90}Sr and ^{90}Y are in secular equilibrium.

(a) What will be the total activity of the sample 10 days later?

Using the general case for serial radioactive decay but modifying it for an initial amount of daughter activity:

$$N_Y = \frac{\lambda_{Sr} N_{Sr}^0}{\lambda_Y - \lambda_{Sr}}\left(e^{-\lambda_{Sr}t} - e^{-\lambda_Y t}\right) + N_Y^0 e^{-\lambda_Y t}$$

Multiplying the equation by λ_Y allows for the equation to be written in activities:

$$\begin{aligned}
A_Y &= \frac{\lambda_Y A_{Sr}^0}{\lambda_Y - \lambda_{Sr}}\left(e^{-\lambda_{Sr}t} - e^{-\lambda_Y t}\right) + A_Y^0 e^{-\lambda_Y t} \\
&= \frac{3.008x10^{-6} \cdot 1x10^9}{3.008x10^{-6} - 7.55x10^{-10}}\left(e^{-7.55x10^{-10}\cdot 10\cdot 24\cdot 3600} - e^{-3.008x10^{-6}\cdot 10\cdot 24\cdot 3600}\right) \\
&+ 0.62x10^9 e^{-3.008x10^{-6}\cdot 10\cdot 24\cdot 3600} \\
&= 0.97GBq
\end{aligned}$$

$$\begin{aligned}
A_{Sr} &= A_{Sr}^0 e^{-\lambda t} \\
&= 1x10^9 e^{-7.55x10^{-10}\cdot 10\cdot 24\cdot 3600} \\
&\approx 1GBq
\end{aligned}$$

$$\therefore A_{total} = 1 + .97GBq = 1.97GBq$$

(b) *What will be the total activity of the sample 29.12 years later?*

$$A_Y = \frac{\lambda_Y A_{Sr}^0}{\lambda_Y - \lambda_{Sr}}\left(e^{-\lambda_{Sr} t} - e^{-\lambda_Y t}\right) + A_Y^0 e^{-\lambda_Y t}$$
$$= \frac{3.008x10^{-6} \cdot 1x10^9}{3.008x10^{-6} - 7.55x10^{-10}}\left(e^{-7.55x10^{-10} \cdot 29.16 \cdot 365 \cdot 24 \cdot 3600} - e^{-3.008x10^{-6} \cdot 29.12 \cdot 365 \cdot 24 \cdot 3600}\right)$$
$$+ 0.62x10^9 e^{-3.008x10^{-6} \cdot 29.12 \cdot 365 \cdot 24 \cdot 3600}$$
$$= 0.5GBq$$

$$A_{Sr} = A_{Sr}^0 e^{-\lambda t}$$
$$= 1x10^9 e^{-7.55x10^{-10} \cdot 29.12 \cdot 365 \cdot 24 \cdot 3600}$$
$$= 0.5GBq$$

$$\therefore A_{total} = 0.5 + 0.5GBq = 1GBq$$

4). (2 points) The global inventory of ^{14}C is about 8.5 x 10^{18} Bq and we are told that all the activity comes from cosmic rays interaction. The activity tells us the number of atoms decaying per second. We can convert that to the mass of ^{14}C that is decaying per second and multiply by a year to determine the mass per year that is decaying and needs to be replenished to keep the activity steady:

$$8.5x10^{18} \frac{atoms}{\sec} \cdot \frac{1mol}{6.023x10^{23} atoms} \cdot \frac{14g}{1mol} \cdot 3600 \cdot 24 \cdot 365 = 6.02kg$$

5). (4 points) Calculate the number of grams contained in a 5 mCi source of the following nuclides.

For each atom, we will calculate the specific activity and then use it to find the weight of each atom:

$$SA = \frac{6.02x10^{23} \lambda}{M} = \frac{4.17x10^{23}}{M \cdot t_{1/2}}$$

(a) ^{18}F, $t_{1/2}$ = 109.77 min

$$SA == \frac{4.17x10^{23}}{18 \cdot 109.77 \cdot 60} = 3.5x10^{18} Bq \cdot g^{-1} = 9.5x10^7 Ci$$

$$\# grams = \frac{g}{9.5x10^{7}Ci} \cdot 5x10^{-3}Ci = 5.25x10^{-11}g$$

(b) ^{14}C, $t_{1/2}$ = 5730 y

$$SA == \frac{4.17x10^{23}}{14 \cdot 5730 \cdot 365 \cdot 24 \cdot 3600} = 1.65x10^{11}Bq \cdot g^{-1} = 4.45Ci$$

$$\# grams = \frac{g}{4.45Ci} \cdot 5x10^{-3}Ci = 0.0011g$$

(c) ^{32}P, $t_{1/2}$ = 14.29 days

$$SA == \frac{4.17x10^{23}}{32 \cdot 14.29 \cdot 24 \cdot 3600} = 1.055x10^{16}Bq \cdot g^{-1} = 2.85x10^{5}Ci$$

$$\# grams = \frac{g}{2.85x10^{5}Ci} \cdot 5x10^{-3}Ci = 1.75x10^{-8}g$$

(d) ^{235}U, $t_{1/2} = 7.038\text{x}10^{8}$ y

$$SA == \frac{4.17x10^{23}}{235 \cdot 7.038x10^{6} \cdot 365 \cdot 24 \cdot 3600} = 8x10^{6}Bq \cdot g^{-1} = 2.16x10^{-4}Ci$$

$$\# grams = \frac{g}{2.16x10^{-4}Ci} \cdot 5x10^{-3}Ci = 23.14g$$

The idea is that even though all four atoms have the same number of radioactive nuclei, they all weigh differently.

6). (3 points) Charcoal found in a deep layer of sediment in a cave is found to have an atomic $^{14}C/^{12}C$ ratio only 30% that of a sample from a higher level with a known age of 1850 years. What is the age of the deeper layer?

^{12}C is stable, but ^{14}C decays with a half-life of 5730 yrs. The exponential decay law is:

$$N = N_0 e^{-\lambda t}$$

In this case we are given a ratio of the number of ^{14}C to ^{12}C, but since ^{12}C is stable the number of atoms will not change, only the number of atoms of ^{14}C will. Therefore we can write the equation as:

$$\frac{N_{^{14}C}}{N_{^{12}C}} = \frac{N^0_{^{14}C}}{N^0_{^{12}C}} e^{-\lambda_{^{14}C} t}$$

We assume that the upper and lower layer of the sediment will have the same initial amount of ^{14}C to ^{12}C ratio upon formation. Calculating then the ratio of the number of atoms for the upper sediment:

$$\frac{N_{^{14}C}}{N_{^{12}C}} = e^{-\frac{\ln 2}{5730} \cdot 1850} = 0.80$$

We are told that the lower level has a 30% of the upper level atomic ratio, therefore (0.80)*(0.30) =0.24 of the ratio of the original amount upon formation. Calculating now for time:

$$0.24 = e^{-\frac{\ln 2}{5730} \cdot t} \rightarrow t = 11802 y$$

7. (5 points) Technicium 99m (^{99m}Tc) is used for nuclear medicine imaging procedures. The minimum dose that generates a useful image is 5 mCi. Generally, each patient receives 10-15 mCi for an imaging procedure. ^{99m}Tc is produced by elution from "generators" containing the bound parent isotope ^{99}Mo. A radioisotope generator is calibrated to contain 500 mCi of ^{99}Mo at 12 noon on Friday, Sept 22. The generator containing ^{99}Mo is delivered on Monday morning at 6 am.

(a) The ^{99m}Tc is eluted at 8 am on Monday, Sept 25. How much ^{99m}Tc is eluted (in mCi)?

This is the decay chain:

$$^{99}Mo \xrightarrow{t_{1/2}=65.94h} {}^{99m}Tc \xrightarrow{t_{1/2}=6.02h} {}^{99}Tc$$

Lets use the general case to solve for the amount of ^{99m}Tc on Monday at 8 AM, a total of 68 hours after the generator is calibrated. The initial activity amount of ^{99m}Tc is zero. Also, λ_{Mo}=2.92x10^{-6} s^{-1} and λ_{99mTc}=3.20x10^{-5} s^{-1}. Also the activity converted to Bq: 1.85x10^{10} s^{-1}. Solving:

$$A_{^{99m}Tc} = \frac{\lambda_{^{99m}Tc} A_{Mo}^0}{\lambda_{^{99m}Tc} - \lambda_{Mo}} \left(e^{-\lambda_{Mo} t} - e^{-\lambda_{^{99m}Tc} t} \right)$$

$$= \frac{3.2x10^{-5} \cdot 1.85x10^{10}}{3.2x10^{-5} - 2.92x10^{-6}} \left(e^{-2.92x10^{-6} \cdot 68 \cdot 3600} - e^{-3.2x10^{-5} \cdot 68 \cdot 3600} \right)$$

$$= 9.9x10^9\, Bq = 267 mCi$$

(b) The generator is then eluted on Wed, Sept 27 at 8 am. Is there enough ^{99m}Tc available for 10 patient imaging studies?

First we need to calculate how much Mo is now left at this time, 68 hours later:

$$A = A_0 e^{-\lambda t}$$

$$= 500 mCi \cdot e^{-2.92x10^{-6} \cdot 68 \cdot 3600} = 244.6 mCi = 9.05x10^9\, Bq$$

This is now our new A_0 for the Mo. Again using the general case, but now the time that has passed is 48 hours:

$$A_{^{99m}Tc} = \frac{\lambda_{^{99m}Tc} A_{Mo}^0}{\lambda_{^{99m}Tc} - \lambda_{Mo}} \left(e^{-\lambda_{Mo} t} - e^{-\lambda_{^{99m}Tc} t} \right)$$

$$= \frac{3.2x10^{-5} \cdot 9.05x10^9}{3.2x10^{-5} - 2.92x10^{-6}} \left(e^{-2.92x10^{-6} \cdot 48 \cdot 3600} - e^{-3.2x10^{-5} \cdot 48 \cdot 3600} \right)$$

$$= 5.97x10^9\, Bq = 161 mCi$$

This will just be enough for 10 patients to receive 15 mCi.

(c) What is the last date that the generator can be used to produce enough ^{99m}Tc for a patient study (i.e., > 5 mCi)?

We need to find the time t when the activity of ^{99m}Tc is 5mCi. Since we are in transient equilibrium we can use equation 4.42 to find the activity of the ^{99}Mo when the activity of ^{99m}Tc is 5 mCi:

$$A_{^{99m}Tc} = \frac{\lambda_{^{99m}Tc} A_{Mo}}{\lambda_{^{99m}Tc} - \lambda_{Mo}}$$

$$1.85x10^8 = \frac{3.2x10^{-5} \cdot A_{Mo}}{3.2x10^{-5} - 2.92x10^{-6}}$$

$$A_{Mo} = 1.68x10^8\, Bq$$

Now solving for time at which this is the activity of the parent:

$$A_{Mo} = A_{Mo}^0 e^{-\lambda t}$$

$$1.68x10^8 = 1.85x10^{10} \cdot e^{-2.92x10^{-6} \cdot t} \Rightarrow t = 18.6 days$$

8. (2 points) The key to this problem is to realize that the parent, ^{238}U is in secular equilibrium with its daughter, ^{226}Ra. With secular equilibrium, the activity of the parent and daughter is equal:

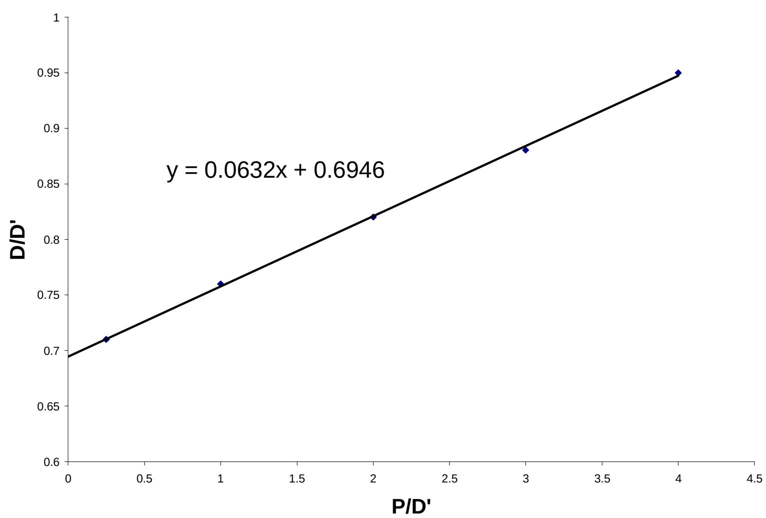
$$\lambda_U N_U = \lambda_{Ra} N_{Ra}$$
$$\lambda_U = \frac{\lambda_{Ra} N_{Ra}}{N_U} = \frac{\ln 2}{1620} \cdot 4.34x10^{-7} = 1.857x10^{-10}\,y$$
$$t_{1/2} = 3.73x10^9\,y$$

Appendix D lists the half life for ^{238}U as $4.468x10^9$ y. Our estimation is 17% off, pretty close using such a simple methodology!

9. (3 points) . Elemental analyses on 5 separate samples from the same meteorite gave the following data, expressed in atoms/sample. How old is this meteorite?

Sample	**^{87}Rb**	**^{87}Sr**	**^{86}Sr**
1	1377	3913	5511
2	16500	12540	16500
3	4050	1661	2025
4	24390	7157	8130
5	16848	4001	4212

Plotting the data:



$$t = \frac{1}{\lambda}\ln\left(1+\frac{\Delta y}{\Delta x}\right) = \frac{1}{\frac{\ln 2}{4.75x10^{10}\,y}}\ln(1+0.0632) = 4.2x10^9\,y$$