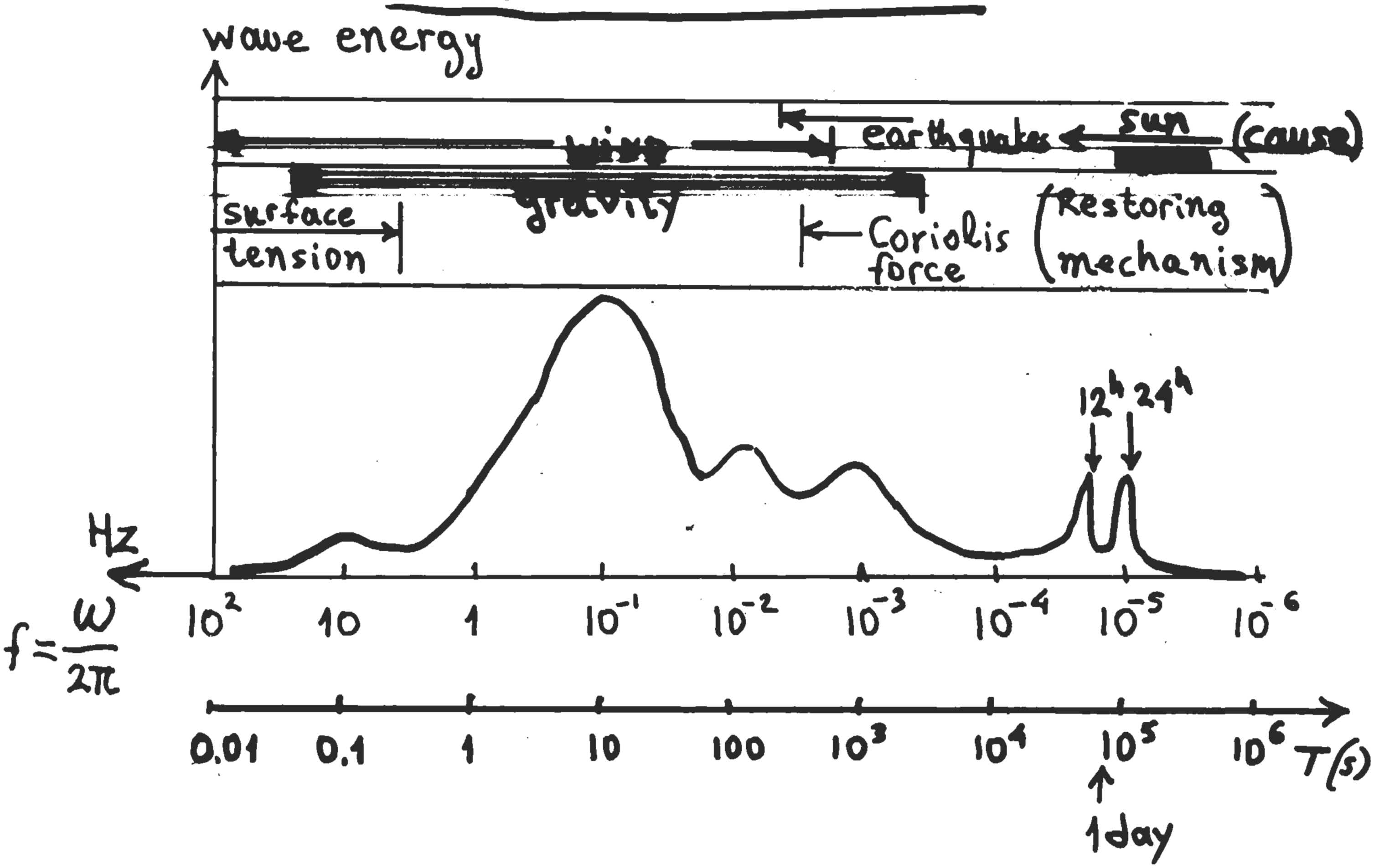
# WAVE SPECTRA





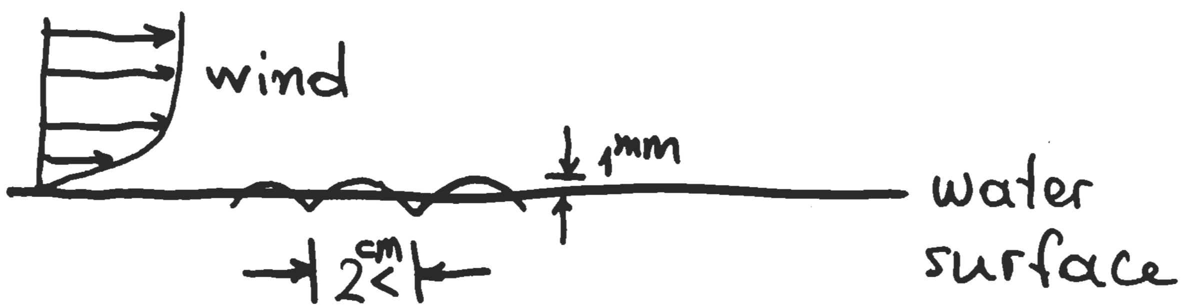

Most waves are wind generated

Gravity / surface tension is principal restoring mechanisms

Earthquake waves (tsunamis) are rare but catastrophic near and on the coast

Tides are slow but can change average sea elevation substantially

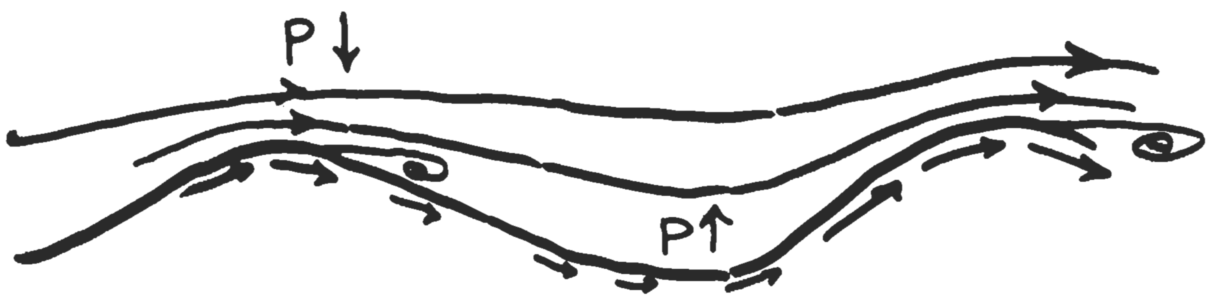
# WAVE GENERATION




→ When wind starts: [0.5 ~2 knots]
Capillary waves form
("cat paws")

→ As wind becomes stronger, waves become longer

$$U_w = 24\ \text{cm/s} \rightarrow \lambda = 1.73\ \text{cm}$$




Bernoulli effect
Frictional drag
Separation drag

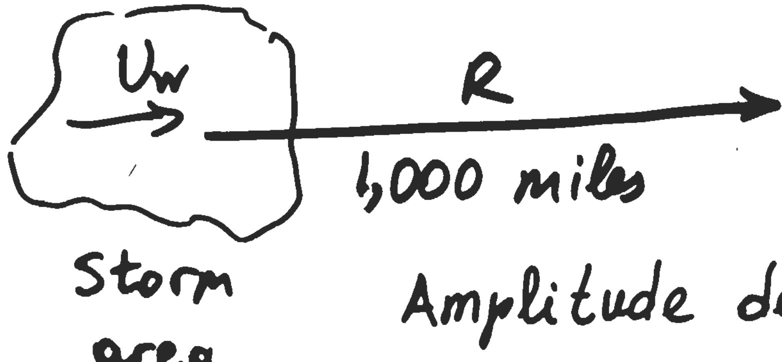

WAVE EVOLUTION
AS WIND BLOWS

Under wind forcing amplitude increases and wavelength increases

$$\lambda \uparrow \Rightarrow k = \frac{2\pi}{\lambda} \downarrow \Rightarrow \omega = \sqrt{kg} \downarrow$$

Once wind stops, viscosity erodes <u>slowly</u> the waves. Small wavelengths decay faster

Number of cycles to reach at a distance R

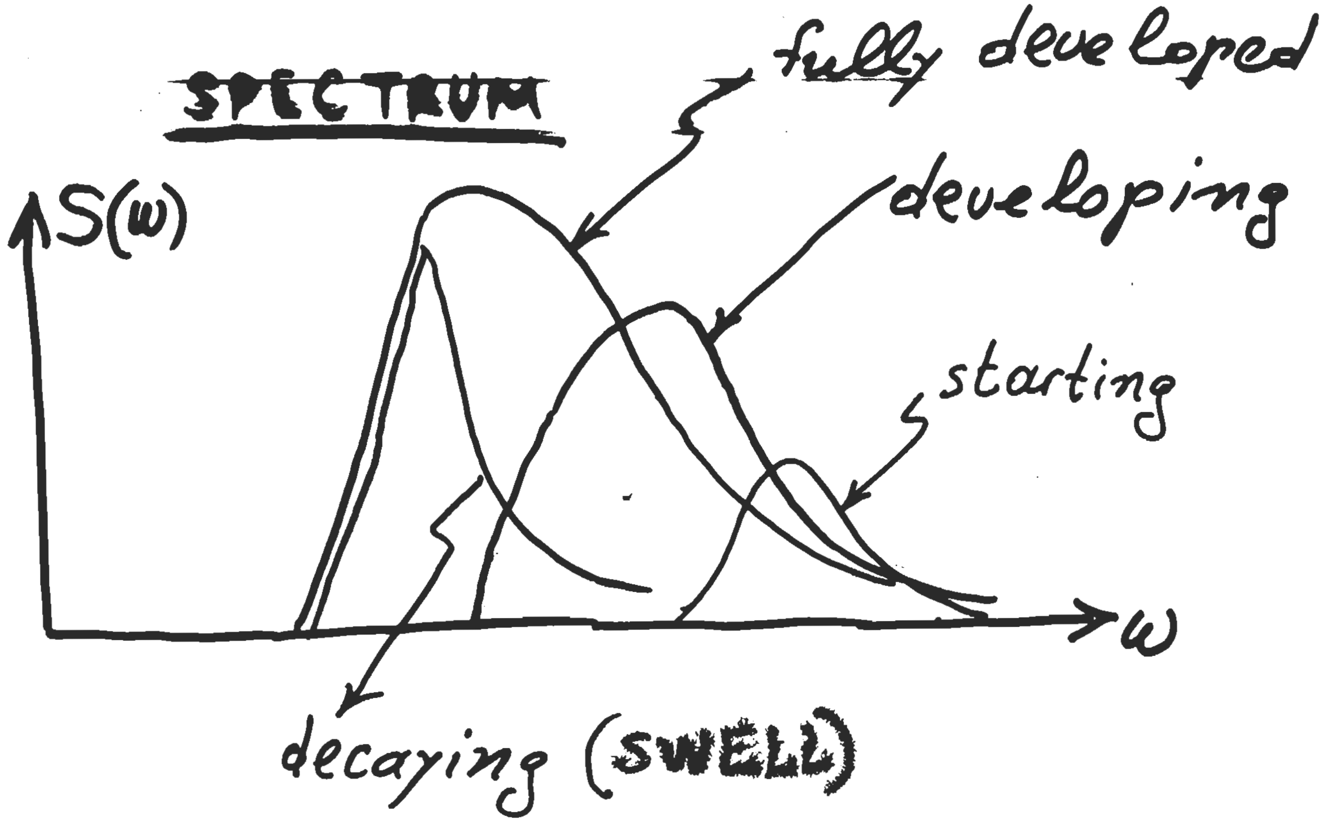



$$N \sim \frac{R}{\lambda}$$

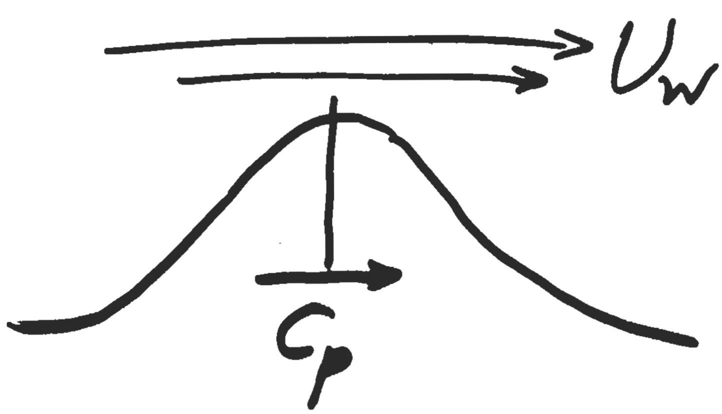
Amplitude decays as $e^{-\gamma t}$

$$\gamma = 2\nu k^2 = 2\nu \omega^4 / g^2$$

## SPECTRUM




Wind must blow over long periods of time and large distances to reach <u>fully developed</u> state




Nonlinear interaction stops (except friction) when

$$U_w \simeq c_p = \frac{\omega}{k} = \frac{g}{\omega}$$

| [redacted] scale | wind speed (mph) | fetch (miles) | duration (h) |
|---|---|---|---|
| 3-4 | 12 | 15 | 3 |
| 5-6 | 25 | 100 | 12 |
| 7 | 35 | 400 | 28 |
| 9 | 50 | 1,050 | 50 |

Required fetch and storm duration

## STANDARD SPECTRA

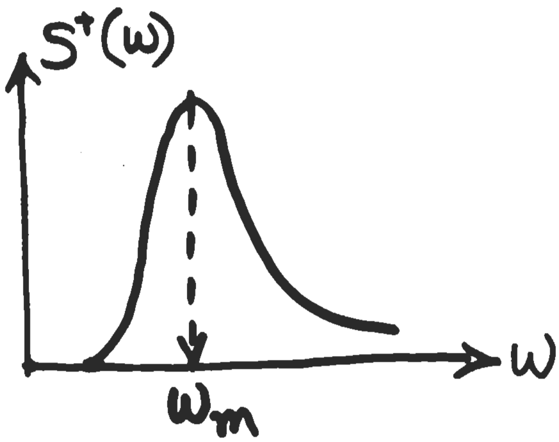
Based on measured spectra and theoretical results, standard forms have been developed.

### Pierson - Moskowitz spectrum

replaced by:

### Bretschneider spectrum

$$S^+(\omega) = \frac{1.25}{4}\,\frac{\omega_m^4}{\omega^5}\,\zeta^2 \exp\left\{-1.25\left(\frac{\omega_m}{\omega}\right)^4\right\}$$




$\omega_m$ = peak or modal frequency,

$\zeta$ = significant wave height

by construction $\int_0^\infty S^+(\omega)\,d\omega = M_0 = \left(\frac{\zeta}{4}\right)^2$

for fully developed seas $\omega_m = 0.4\sqrt{\frac{g}{\zeta}}$

Assumptions used to derive B. spectrum:

Deep water

North Atlantic data

Unlimited fetch

Uni-directional seas

No swell

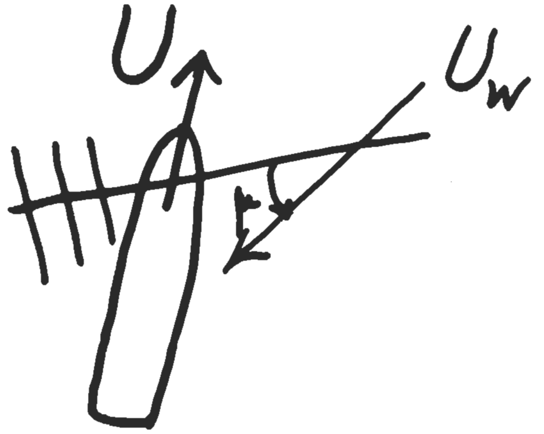
The spectrum is strictly valid for fully developed seas. Developing seas have a broader peak. Decaying seas have a narrower peak


Use two B. spectra to represent local storm + swell

$$S^+(\omega) = S_1^+(\omega) + S_2^+(\omega)$$

To correct for uni-directionality

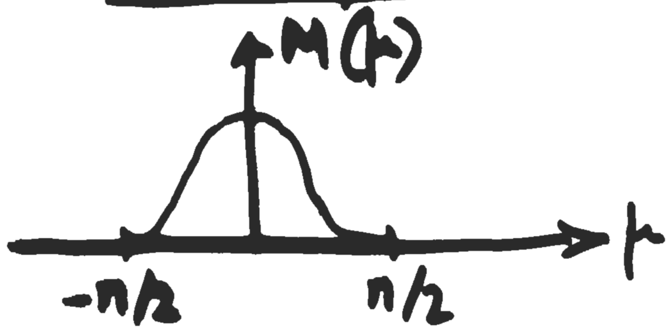
$$S^{+}(\omega,\mu) = \underbrace{S^{+}(\omega)}_{\text{Bretschneider}} \, M(\mu)$$




$M(\mu)$ spreads the energy over a certain angle contained within $(-\pi, \pi)$ from the wind direction

$$\int_{-\pi}^{\pi} M(\mu) = 1$$

Example




$$M(\mu) = \frac{2}{\pi}\cos^2\mu$$

$$-\frac{\pi}{2} < \mu < \pi/2$$

## JONSWAP spectrum

Developed for the limited fetch North Sea by the offshore industry is used extensively

$$S^+(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left\{-\frac{5}{4}\left(\frac{\omega_m}{\omega}\right)^4\right\} \gamma^\delta$$

$$\delta = -\frac{(\omega - \omega_m)^2}{2\sigma^2 \omega_m^2}$$

$$\alpha = 0.076 \, \bar{x}^{-0.22}$$

$$\bar{x} = \frac{gx}{U^2}$$

, $x$ = fetch (n. miles)

$U$ = wind speed (knots)

$$\sigma = \begin{cases} 0.07 & \omega \leq \omega_m \\ 0.09 & \omega > \omega_m \end{cases}$$

$\omega_m$ = peak frequency

$\gamma$ = sharpness parameter

~~Other spectra~~

## Ochi spectrum

$$S^+(\omega) = \frac{1}{4}\,\frac{\left(\frac{4\lambda+1}{4}\,\omega_m^4\right)^{\lambda}}{\Gamma(\lambda)}\,\frac{J^2}{\omega^{4\lambda+1}}\,\exp\left\{-\left(\frac{4\lambda+1}{4}\right)\left(\frac{\omega_m}{\omega}\right)^4\right\}$$

$\lambda$ determines width of spectrum

$\Gamma(x)$ = gamma function of $x$

Ochi spectrum is an extension of the Bretscheider spectrum, allowing to make it wider ($\lambda$ small) for developing seas, or narrower ($\lambda$ large) for swell.

No guidelines on specific $\lambda$ to use.

## IMPORTANT ISSUES

For short term statistics we need moments of spectrum

$$M_n = \int_0^\infty S^+(\omega)\, \omega^n \, d\omega$$

$$n = 0, 2, 4 \ldots$$

From the Bretschneider form

$$S^+(\omega) = \frac{A}{\omega^5} \exp\left\{-B\left(\frac{\omega_m}{\omega}\right)^4\right\}$$

when $\omega >> \omega_m \Rightarrow S^+(\omega) \simeq \dfrac{A}{\omega^5}$

$$M_n = \int_0^{\omega_1} + \int_{\omega_1}^\infty S^+(\omega)\, \omega^n \, d\omega$$

for $M_4 \rightarrow \displaystyle\int_{\omega_1}^\infty S^+(\omega)\omega^4 d\omega \simeq \int_{\omega_1}^\infty \frac{A}{\omega} d\omega$

$$= A \left[\log_e \omega\right]_{\omega_1}^\infty$$

~~In application~~

$$S_y(\omega) = |H(\omega)|^2 S(\omega)$$

$H(\omega)$ always $\to 0$ as $\omega \to \infty$, so there is no physical problem (same applies to waves). There may be a mathematical problem, though.

Two solutions:

(1) Truncate integration up to $3 \cdot \omega_m$. This has the advantage that scaling can be applied between model – full scale tests

(2) Choose constant upper limit, typically $\omega_{max} = 2.0$ rad/s.
When scaling this may be necessary to avoid very large testing