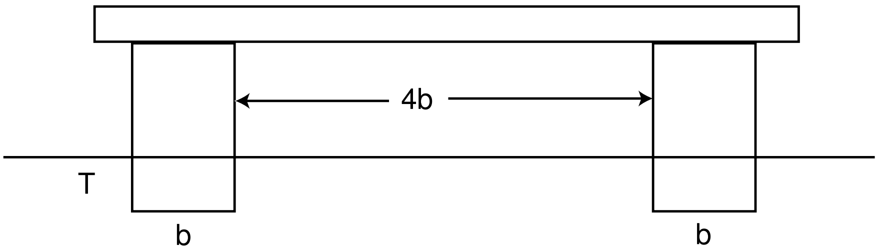
Massachusetts Institute of Technology
Subject 2.017 Design of Systems for Uncertain Environments

DESIGN WORK 4

Assigned 16 March 2006
Due 23 March 2006

1. **Metacentric Height**. Consider the roll stability of a catamaran with cross-section as shown in the figure.
   (a) What is the distance KM for this section, at small roll angles? You may approximate the area of the "buoyancy wedges" as the mean height times the width.
   (b) What is KM for a single block of draft T and beam $2b$?
   (c) Make a sketch of the righting moment as the angle increases from zero to the point where one hull lifts out of the water, and then beyond. Describe what has happened in this situation and hence one of the inherent problems with catamarans.



2. **Simulation of a system driven by a random disturbance.**
   (a) Simulate the second-order system (written three ways here):

$$
\begin{aligned}
x'' + ax' + bx &= d(t) \\
x'' + 2\zeta\omega_n x' + \omega_n^2 x &= d(t) \\
\frac{d}{dt}\left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} &= \left[\begin{array}{cc} -a & -b \\ 1 & 0 \end{array} \right] \left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} + \left[\begin{array}{c} d(t) \\ 0 \end{array} \right],
\end{aligned}
$$

with $a = 1$ and $b = 1$. We are interested in its response to the following disturbance $d(t)$, which you see is added onto the right-hand side of the equations.

$$d(t) = \sum_{i=1}^{9} A_i \sin(\omega_i t + \phi_i), \text{ where}$$

i	A_i	ω_i
1	0.2	0.50
2	0.4	0.75
3	0.6	1.00
4	0.6	1.25
5	0.5	1.50
6	0.4	1.75
7	0.3	2.00
8	0.2	2.25
9	0.1	2.50

Notes for MATLAB: You can set each phase angle to a random value in the range $[0, 2\pi]$ with the command `phi(i) = 2*pi*rand;`. Compute d inside the derivative function - it is purely a function of time. Use the initial condition $[0, 0]$, and final time sixty seconds. Be sure to show a labelled plot of time vs. x and time vs. d. You have just simulated the roll motion of a (badly-designed) surface vessel in waves!

(b) From the graph, about what is the "significant height" of the roll motion?

(c) What is the effect of reducing or increasing the damping in this system, say $a = 0.5$ and then $a = 4$?

3. **Feedback control design.** The directional behavior of a highly maneuverable vessel is modelled by

$$\frac{\theta}{\delta} = \frac{0.1s + 1}{s^2 + 0.6s - 0.05}$$

where the rudder deflection is δ and the heading is θ. You note that because the transfer function denominator has a negative coefficient, the system is unstable without a control system. Also, the numerator shows that the hydrodynamic lift on the rudder has a component that scales rudder position as well as one that scales rudder rotation *rate*.

(a) Find the *poles* (the roots of the denominator polynomial) of this system. Are the roots complex (indicating an oscillatory behavior), or real (indicating two exponential growth/decay modes)? At what rate does the system go unstable - that is, over what time does the response grow by a factor of e?

You can confirm this with a simulation of the system behavior from some nonzero initial condition.

(b) Design a proportional-derivative controller for the boat (PD), so as to achieve a closed-loop bandwidth of $\omega_c = 1$ rad/s and a damping ratio of $\zeta = 0.5$. The idea here is to choose k_p and k_d so as to put the closed-loop poles - that is, the roots of $1 + P(s)C(s) = 0$ - at the specific locations $\omega_c\zeta \pm j\omega_c\sqrt{1-\zeta^2}$. The easiest way to carry this out is to write out the polynomial with k_d and k_p as free variables, and recognize that this same polynomial can be written as $s^2 + 2\zeta\omega_c s + \omega_c^2 = 0$. Some algebra will give you a set of two equations in two unknowns.

(c) Simulate the plant and controller, combined as a negative feedback system, in a step response, and show a plot. Note the step is applied to the commanded position; so you have to combine the plant and the controller, using $\delta = (k_p + sk_d)(\theta_{command} - \theta)$. Does the response have the time scale and damping properties that you designed for?

(d) After the transients of the step response have died out, what is the steady-state error?