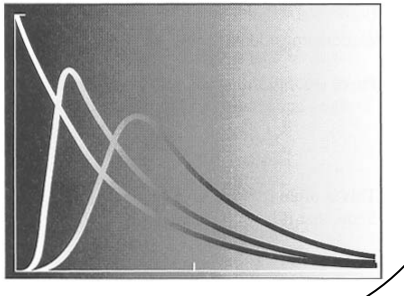
Signals and Systems

Fall 2003

Lecture #3

11 September 2003

1) Representation of DT signals in terms of shifted unit samples
2) Convolution sum representation of DT LTI systems
3) Examples
4) The unit sample response and properties of DT LTI systems



Exploiting Superposition and Time-Invariance

$$x[n] = \sum_k a_k x_k[n] \xrightarrow{LinearSystem} y[n] = \sum_k a_k y_k[n]$$

Question: Are there sets of "basic" signals so that:

a) We can represent rich classes of signals as linear combinations of these building block signals.

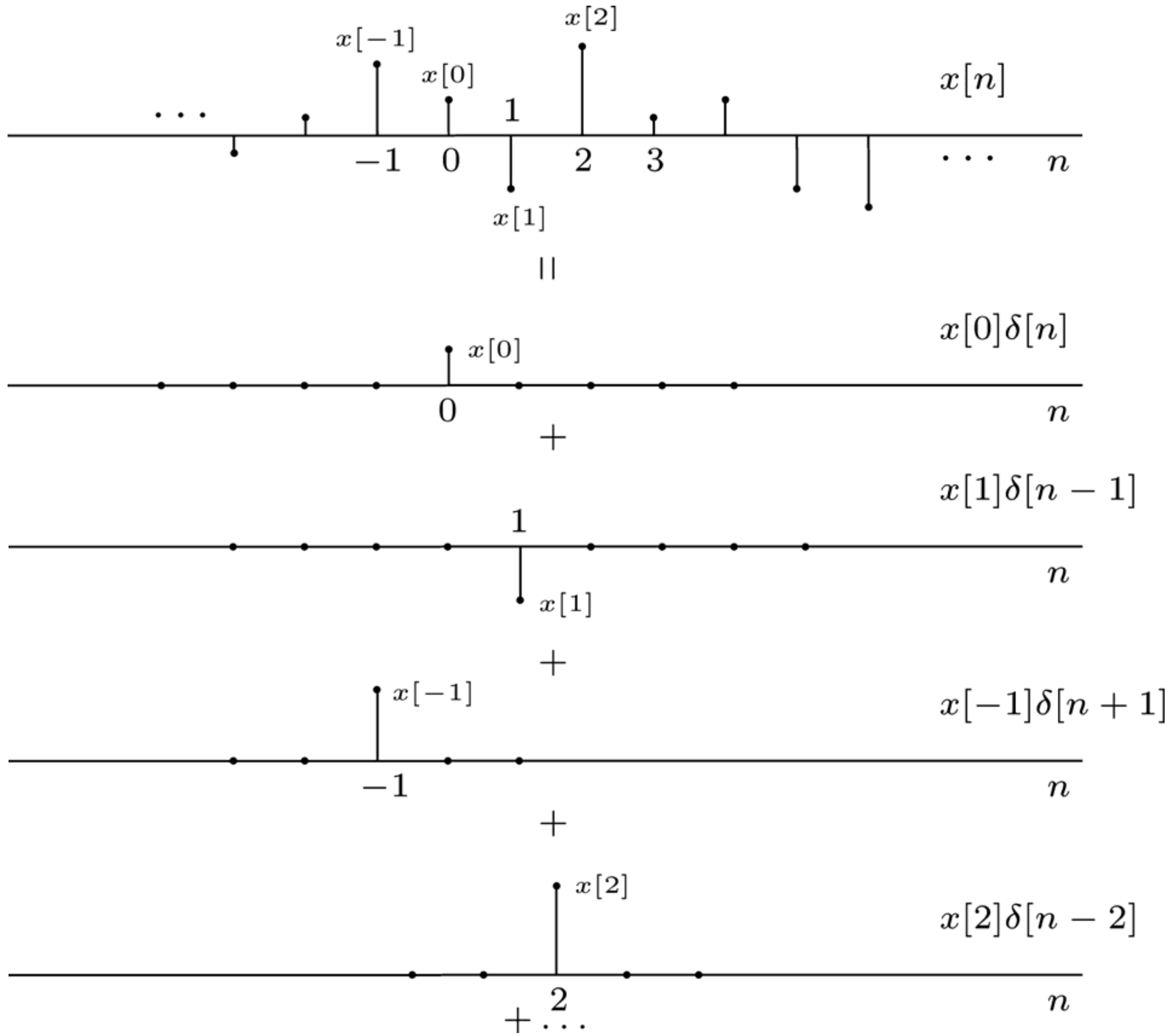
b) The response of LTI Systems to these basic signals are both *simple* and *insightful*.

Fact: For LTI Systems (CT or DT) there are two natural choices for these building blocks

Focus for now:

	DT	Shifted unit samples
	CT	Shifted unit impulses

Representation of DT Signals Using Unit Samples



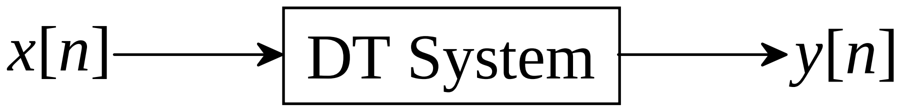
That is ...

$$x[n] = \cdots + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \cdots$$

$$\Downarrow$$

$$x[n] = \sum_{k=-\infty}^{\infty} \underbrace{x[k]}_{\text{Coefficients}} \underbrace{\delta[n-k]}_{\text{Basic Signals}}$$

The Sifting Property of the Unit Sample

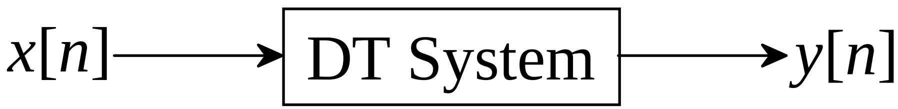


- Suppose the system is **linear**, and define $h_k[n]$ as the response to $\delta[n - k]$:

$$\delta[n-k] \rightarrow h_k[n]$$

From superposition: $$\Downarrow$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h_k[n]$$



- Now suppose the system is **LTI**, and define the *unit sample response h*[*n*]:

$$\delta[n] \rightarrow h[n]$$

From TI:

$$\Downarrow$$

$$\delta[n-k] \rightarrow h[n-k]$$

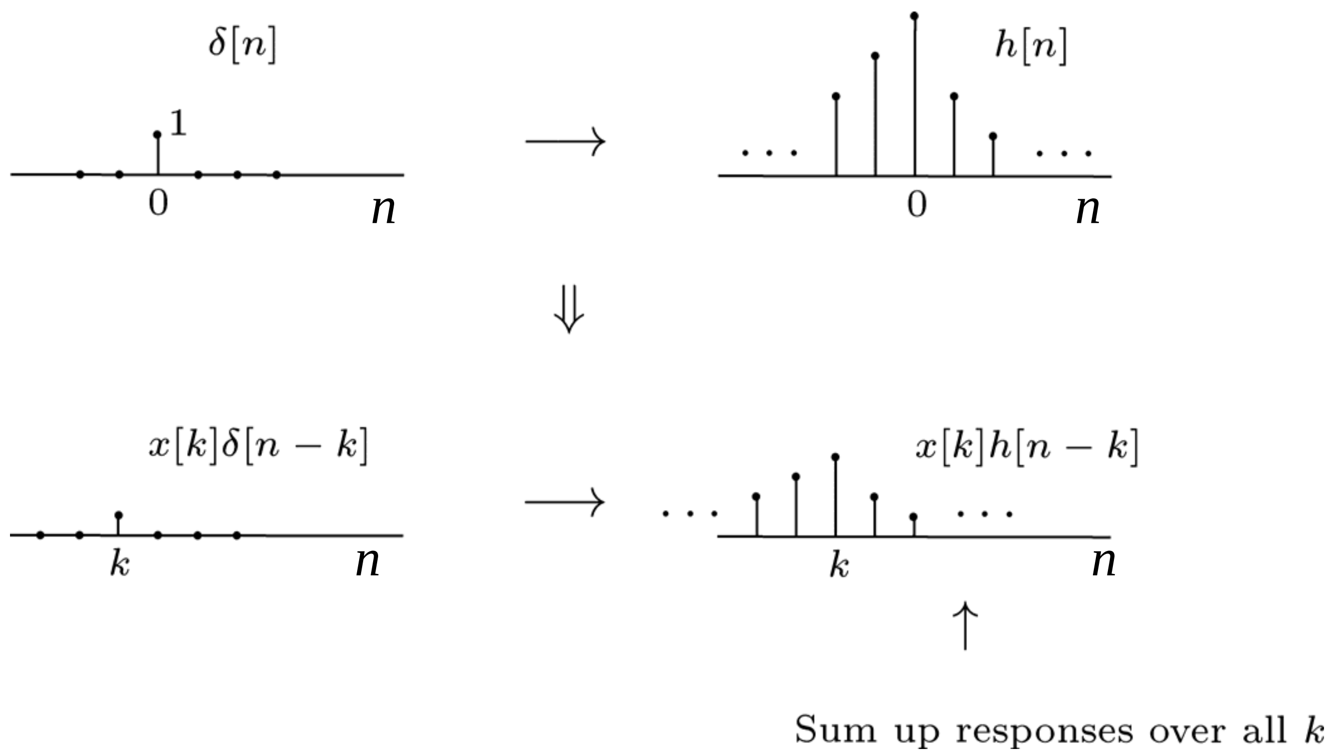
From LTI:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \rightarrow \underbrace{y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]}_{\text{Convolution Sum}}$$

Convolution Sum Representation of Response of LTI Systems

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Interpretation

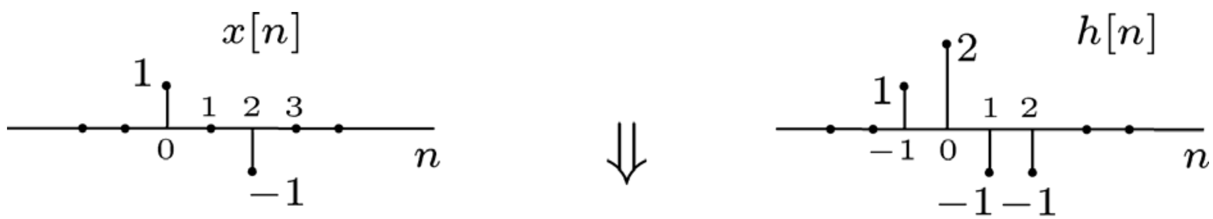


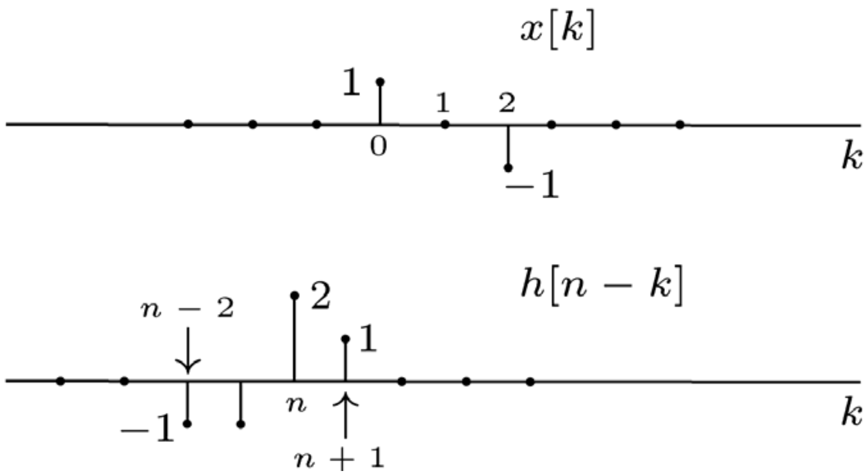
Visualizing the calculation of $y[n] = x[n] * h[n]$

Choose value of *n* and consider it fixed

$$y[n] = \sum_{k=\infty}^{\infty} x[k]h[n-k]$$

View as functions of *k* with *n* fixed

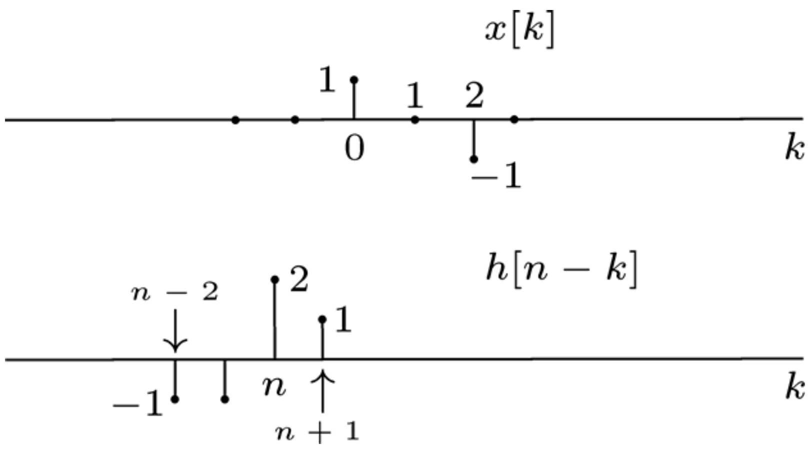




***y*[0] = ∑ prod of overlap for *n* = 0**

***y*[1] = ∑ prod of overlap for *n* = 1**

Calculating Successive Values: Shift, Multiply, Sum



$$
\begin{aligned}
y[n] &= 0 \quad \text{for } n < \mathbf{-1} \\
y[-1] &= \mathbf{1 \times 1 = 1} \\
y[0] &= \mathbf{0 \times 1 + 1 \times 2 = 2} \\
y[1] &= \mathbf{(-1) \times 1 + 0 \times 2 + 1 \times (-1) = -2} \\
y[2] &= \mathbf{(-1) \times 2 + 0 \times (-1) + 1 \times (-1) = -3} \\
y[3] &= \mathbf{(-1) \times (-1) + 0 \times (-1) = 1} \\
y[4] &= \mathbf{(-1) \times (-1) = 1} \\
y[n] &= 0 \quad \text{for } n > \mathbf{4}
\end{aligned}
$$

Properties of Convolution and DT LTI Systems

1) A DT LTI System is *completely characterized* by its unit sample response

Ex. #1: $h[n] = \delta[n - n_0]$

There are *many* systems with this repsonse to $\delta[n]$

There is only *one* LTI System with this response to $\delta[n]$:

$$y[n] = x[n - n_0]$$

$$\Downarrow$$

$$\boxed{x[n] * \delta[n - n_0] = x[n - n_0]}$$

Ex. #2:

$y[n] = \sum_{k=-\infty}^{n} x[k]$ - An Accumulator

Unit Sample response

$$h[n] = \sum_{k=-\infty}^{n} \delta[k] = u[n]$$

$$\Downarrow$$

$$x[n] * u[n] = \sum_{k=-\infty}^{n} x[k]$$

The Commutative Property

$$y[n] = x[n] * h[n] = h[n] * x[n]$$

<u>Ex</u>: Step response $s[n]$ of an LTI system

$$s[n] = u[n] * h[n] = h[n] * u[n]$$

step input

“input”

Unit Sample response of accumulator

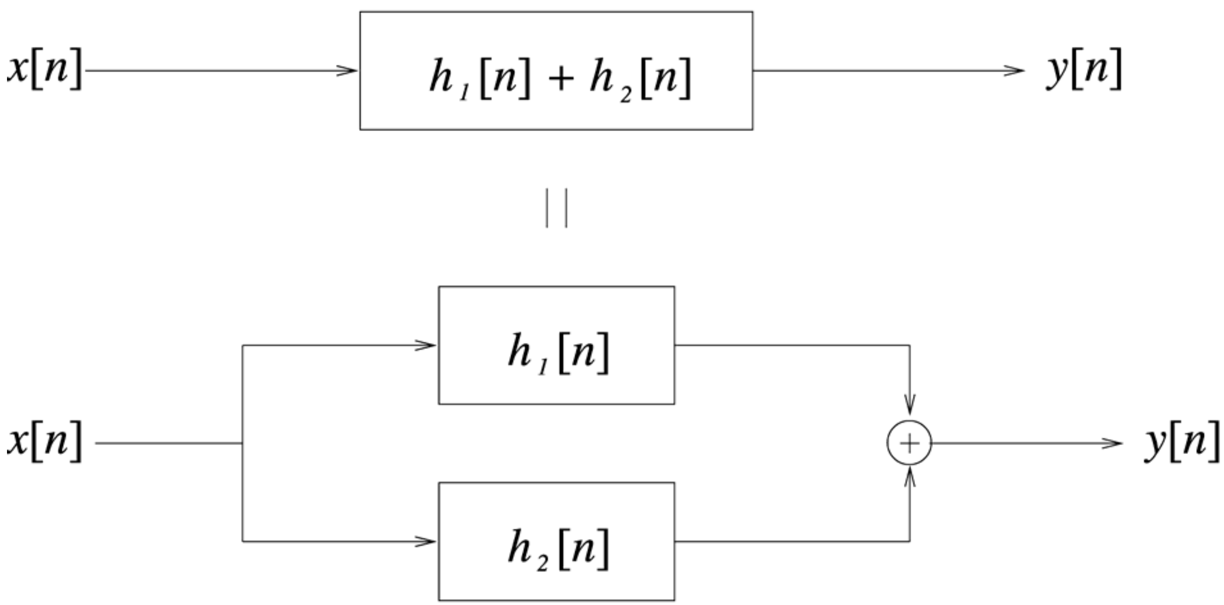
$$\Downarrow$$

$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

The Distributive Property

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x * h_2[n]$$

Interpretation



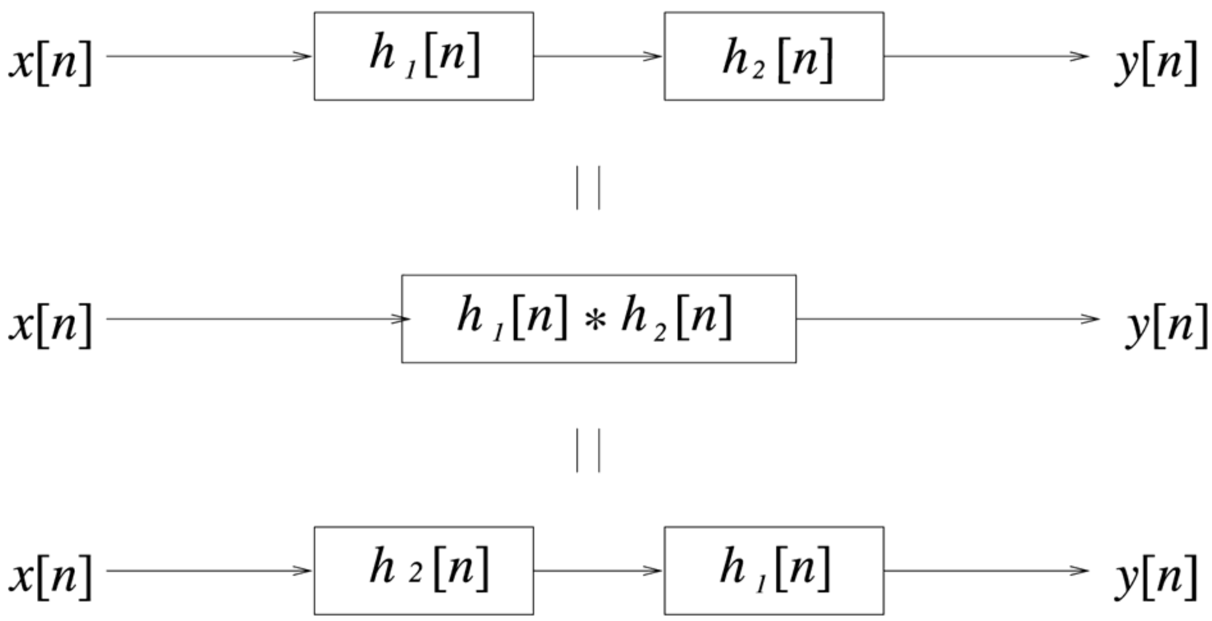
The Associative Property

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$$

(Commutativity) $\|$

$$x[n] * (h_2[n] * h_1[n]) = (x[n] * h_2[n]) * h_1[n]$$

Implication (Very special to LTI Systems)



Properties of LTI Systems

1) Causality $\Leftrightarrow$ $h[n] = 0 \text{ for all } n < 0$

2) Stability $\Leftrightarrow$ $$\sum_{k=-\infty}^{+\infty} |h[k]| < \infty$$