

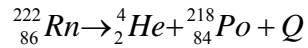
22.01 Problem Set 2 Homework Solutions

(* Good website to check out for decay schemes: <http://ie.lbl.gov/education/isotopes.htm>)

(* Good website to get isotope explorer: <http://ie.lbl.gov/toi.html>)

(* Good website to get exact atomic masses and mass excess:
<http://t2.lanl.gov/data/astro/molnix96/massd.html>)

3.12 (3 points) The decay reaction:



a). The energy, Q, released in the decay is the difference in the masses between the parent nucleus and the daughter nuclei. Therefore, this mass difference can be expressed using the mass excesses, and can be written:

$$\begin{aligned} Q &= \Delta_{\text{parent}} - \Delta_{\text{daughter}} - \Delta_{\alpha} & (3.13) \\ &= \Delta_{\text{Rn}} - \Delta_{\text{Po}} - \Delta_{\alpha} \\ &= 16.93 - 8.38 - 2.4248 \\ &= 5.5852 \text{ MeV} \end{aligned}$$

(* Note, this energy is shared between the alpha particle and the daughter nucleus)

b). The energy of the alpha particle is determined by using the conservation of momentum and of energy. Section 3.3 in the text has a good description and proof of the following formula:

$$E_{\alpha} = \frac{MQ}{m + M} \quad (3.18)$$

where M is the mass of the recoil daughter, and m is the mass of the alpha particle.

Therefore, for the Radon decay:

$$E_{\alpha} = \frac{(218)(5.5852)}{4 + 218} = 5.485 \text{ MeV}$$

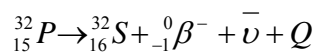
Notice that most of the energy is given to the daughter nucleus.

c). Using conservation of energy:

$$E_{\text{recoil}} = Q - E_{\alpha} = 5.5852 - 5.485 = .1002 \text{ MeV}$$

3.17 (2 points)

a). The beta minus decay equation is:

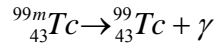


$$\begin{aligned} Q_{\beta^{-}} &= \Delta_P - \Delta_D \\ &= -24.303 - (-26.013) = 1.71 \text{ MeV} \end{aligned}$$

b). The Q-value energy, the energy released from the decay, is shared between the antineutrino, the beta particle, and the recoil ^{32}S atom, but the ^{32}S atom is going to receive a very negligible amount of the decay energy. Therefore:

$$E_\nu = Q - E_{\beta^-} = 1.71 - 0.65 = 1.06 \text{ MeV}$$

3.28 (# points) The decay equation:



We know that the initial momentum is zero, so the momentum of the gamma and the momentum of ${}^{99}\text{Tc}$ has to be the same, and the kinetic energy of ${}^{99}\text{Tc}$ is:

$$E_{{}^{99}\text{Tc}} = \frac{1}{2} m_{{}^{99}\text{Tc}} v_{{}^{99}\text{Tc}}^2$$

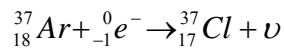
$$p_\gamma = p_{{}^{99}\text{Tc}} = m_{{}^{99}\text{Tc}} v_{{}^{99}\text{Tc}} \rightarrow v_{{}^{99}\text{Tc}} = \frac{p_{{}^{99}\text{Tc}}}{m_{{}^{99}\text{Tc}}} = \frac{p_\gamma}{m_{{}^{99}\text{Tc}}}$$

From Appendix D, the gamma's energy is 140 keV. Solving for the energy of ${}^{99}\text{Tc}$:

$$E_{{}^{99m}\text{Tc}} = \frac{1}{2} m_{{}^{99m}\text{Tc}} \left(\frac{p_\gamma}{m_{{}^{99m}\text{Tc}}} \right)^2 = \frac{hc}{2m_{{}^{99m}\text{Tc}}c^2} = \frac{(140)^2}{2 \cdot 92131.37} = .106 \text{ eV}$$

1) (# points)

a). The decay equation is:



Neglecting the binding energy of the electron:

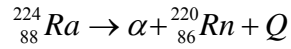
$$\begin{aligned} Q_{\text{EC}} &= \Delta_{\text{parent}} - \Delta_{\text{daughter}} & (3.25) \\ &= \Delta_{\text{Ar}} - \Delta_{\text{Ni}} \\ &= -30.951 - (-31.765) \\ &= 0.814 \text{ MeV} \end{aligned}$$

b). Including now the electron binding energy 3.2 keV = .0032 MeV gives:

$$\begin{aligned} Q_{\text{EC}} &= \Delta_{\text{parent}} - \Delta_{\text{daughter}} - E_{\text{B}} \\ &= 0.814 - 0.0032 \\ &= 0.8108 \text{ MeV} \end{aligned}$$

c). The neutrino will take the entire energy, the Q energy, from the reaction since it is practically mass less.

2. (# points) The decay equation for the alpha decay is:



Calculating the Q value of the reaction, written with masses in amu:

$$\begin{aligned} Q &= M_{\text{parent}} - M_{\text{daughter}} - M_{\alpha} & (3.12) \\ &= M_{\text{Ra}} - M_{\text{Rn}} - M_{\alpha} \\ &= 224.020202 - 220.011384 - 4.00260305 \\ &= 0.00621495 \text{ amu} \\ &= 5.789 \text{ MeV} \end{aligned}$$

The energy of the alpha particle is:

$$E_{\alpha} = \frac{MQ}{m + M}$$

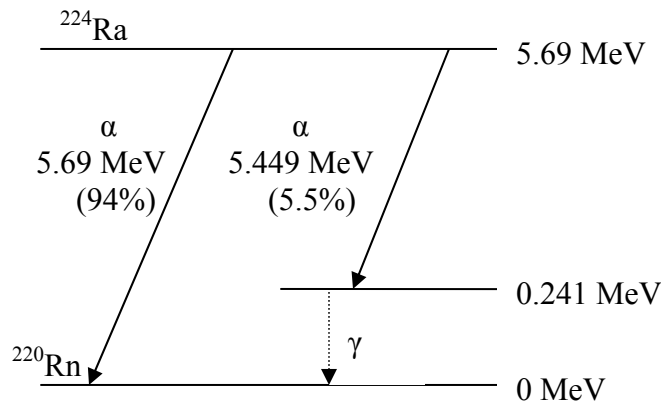
This is the maximum energy available for the daughter, ${}^{220}\text{Rn}$ going to the ground state. Calculating the energy of the alpha particle gives:

$$E_{\alpha} = \frac{220(5.789)}{4 + 220} = 5.69 \text{ MeV}$$

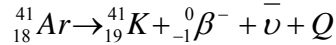
The other less frequent alpha occurring 5.5% of the time leaves the daughter nucleus in an excited state of 0.241 MeV above the ground state. Therefore the energy of this alpha is:

$$E_{\alpha} = 5.69 - 0.241 = 5.449 \text{ MeV}$$

The nuclear decay scheme that shows this is:



3. (# points) The decay equation is:

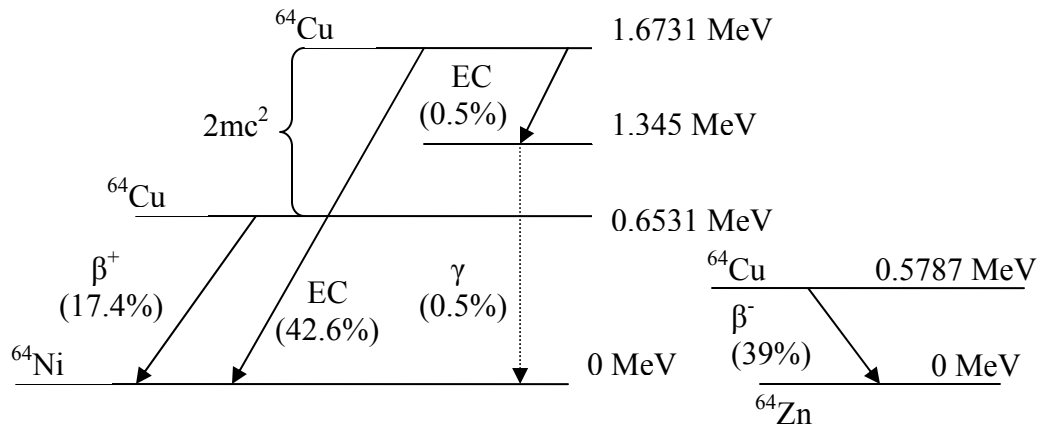


Calculating the Q energy, using masses:

$$\begin{aligned} Q &= M_{\text{parent}} - M_{\text{daughter}} \\ &= M_{\text{Ar}} - M_{\text{K}} \\ &= 40.964500 - 40.961825 \\ &= 0.002675 \text{ amu} \\ &= 2.492 \text{ MeV} \end{aligned}$$

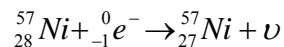
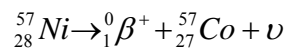
The beta emission here leaves the daughter ${}^{41}\text{K}$ atom in an excited state of 1.293 MeV. Therefore the remaining energy will go to the beta minus and the antineutrino, hence 1.2 MeV. The maximum energy that the emitted beta minus will have is then the total remaining energy with the antineutrino receiving nothing, therefore the maximum kinetic energy is 1.2 MeV.

4. (# points) The decay scheme is:



3.26 NOT GRADED

a). Calculating the Q value for the two decays:



$$\begin{aligned}
 Q_{\beta^+} &= \Delta_{\text{parent}} - \Delta_{\text{daughter}} - 2mc^2 & (3.41) \\
 &= \Delta_{\text{Ni}} - \Delta_{\text{Co}} - 1.022 \\
 &= -56.10 - (-59.339) - 1.022 \text{ MeV} \\
 &= 2.217 \text{ MeV}
 \end{aligned}$$

$$Q_{\text{EC}} = \Delta_{\text{parent}} - \Delta_{\text{daughter}} - E_B \quad (3.25)$$

First, you need to calculate E_B , which is the electron binding energy in the k-shell. Using the Bohr atomic theory, where $n=1$ and $Z=28$:

$$E_B = \frac{-13.6Z^2}{n^2} \text{ eV}$$

$$E_B = .011 \text{ MeV}$$

$$\begin{aligned}
 Q_{\text{EC}} &= \Delta_{\text{Ni}} - \Delta_{\text{Co}} - .038 \\
 &= -56.10 - (-59.339) - .038 \\
 &= 3.201 \text{ MeV}
 \end{aligned}$$

b). To begin the decay scheme, we know that the maximum energy of decay for the positron decay is 2.217 MeV and for the electron capture decay, it is 3.201 MeV or approximately 1.022 MeV higher than the positron decay path. From Appendix D we find that for ^{57}Ni :

β^+ : 0.843 max (40%), EC (60%)
 γ : 0.127 (13%), 1.38 (78%), 1.76 (7%), 1.92 (15%)
 0.511 keV (80%, γ^\pm) and Co x-rays

In a decay scheme, since the 0.511 keV photons are a production of the annihilation of the positron with an electron and not from anything occurring in the Ni atom, they are not included. The Co x-rays are also not included, but do you know how they are formed? The decay scheme is therefore (with the info from the book it is not possible to know where the 0.127 gamma is – I had to look it up):

