# 16.060 Lecture 19

# State Space Design

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**Today's Topics**

1. Pole Assignment with full state feed back
2. Design with sensor feedback

Possibly the most important implication for controllability, as we have discussed it in the last two lectures, is the assurance it gives us for freedom in the design of feedback control systems. In particular we have

Feedback Control Theorem - Any specified set of closed loop system poles can be obtained by feeding back all of the states, if and only if the systems is controllable.

To achieve this, consider the system

For which *A, B* is a controllable pair. Hence the matrix.

has *n* linearly independent columns

Then, suppose we can determine or measurs all of the states and feed them back so

Hence

or

Which has the block diagram

We want to determine the values for the elements of the feedback gain matrix $K$ so that the resulting system has its $n$ poles located where we want them to be. In other words, for the system

which has the characteristic equation

we want to choose $K$ so the roots are where we want them to be.

The left side of the equation is an nth order polynomial. The coefficients of the various powers of $s$ will be functions of the elements of the $K$ matrix. We will determine those elements so that the characteristic equation has the desired roots.

<u>Example</u>

Design a fall state feedback system for the Quanser. A system block diagram is

The system can also be drawn in the following block diagram form.

with the transfer function

Now suppose we would like the complex pole pair to have an undamped natural frequency of 2 rad/sec. and damping ratio of 0.7 and maintain the motor pole at -6.

The closed loop characteristic function will be

We want this to be the characteristic equation for the closed loop system.

Let's create a state space model for the system. Define

Then

Or

Now we have the three states and one input

So the feedback gain matrix $K$ will have one row and three columns (1x3)

Each element of $K$ will be a feedback gain on a state. For example of $k_2$ will be the feedback gain on state , which is .

Now recall, we want the characteristic equation for the system.

to be

In other words we want

Because then the closed loop system poles will be at the desired locations. Now

Then

And the determinant is

Equating coefficients of the various powers of s to the coefficients in the desired polynomial obtains

These three are readily solved to yield

So the feedback control is

And the system block diagram becomes

Now in most actual design situations it is not possible to have sensor for all the states, so that they can all be fed back to create the control. Typically we have an output vector

which is smaller in dimensions than $\underline{x}$. In such situations it is common to use an estimator to obtain an estimate $\hat{\underline{x}}$ of the state $\underline{x}$.

The estimator would take $\underline{y}$ as its input and produce $\underline{\hat{x}}$ as its output. Then $\hat{x}$ would be multiplied by the gain matrix $K$ to produce the control $\underline{u}$. The system block diagram is then

The question is then- how do we design the estimator so that the total system will satisfy our requirements? Well there is a whole body of knowledge about how to do this. It turns out that for a number of reasons the following form for the estimator works very well

This system has the block diagram

Lets look at each part of this system.

The terms

simulate the actual system. The term

is a feedback which is the difference between the actual sensed quantities ( $\underline{y}$ ) and what the estimator thinks there quantities should be ( $C\hat{x}$ ) , as inferred from the estimate $\underline{\hat{x}}$. The gain matrix

$L$ is then chosen to assure that $\underline{\hat{x}}$ is a good replication of $\underline{x}$. In particular, lets look at the error between $x$ and $\underline{\hat{x}}$

and the D.E. for the error is

or

Now we want $\underline{e}$ to go to zero, implying that we want (*A*-*LC*) to be a stable system. Thus we have a problem similar to the one we just did to assign poles using a feedback gain matrix. In particular the task is to choose the estimator feedback gain *L* so the estimator poles are where we want them.

The total system then looks like this-

The problem then boils down to choosing $K$ and $L$ to satisfy our requirements. It turns out that the poles of the estimator can be assigned any where in the $s$ plane if the system is "observable". Observability is the dual of "controllability". Observability requires that $\Gamma_0$ have $n$ linearly independent columns

This architecture turns out to be very good for implementing aircraft flight management systems. In that case

$A, B \Rightarrow$ linear model of aircraft dynamics

$C \Rightarrow$ GPS and inertial sensors

$\underline{x} \Rightarrow$ position, velocity, acceleration, attitude, attitude rates, (possible additional states such as $\alpha, \beta$ etc.)

$\underline{y} \Rightarrow$ position and velocity from GPS angular velocities and accelerations from the inertial sensors