

PROBLEM SET 5
Due: 10/23/2003

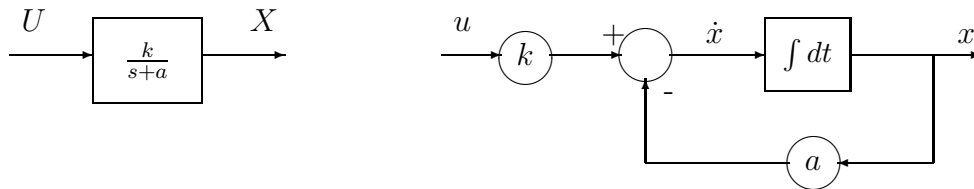
Problem 1: State Space Models for MIMO Systems

State-space models are extremely useful for studying systems with more than one input and/or more than one output. Problem 11.9 in Van de Vegte gives two block diagrams. The first is for a system with two inputs and one output, and the second is for a system with two inputs and two outputs. For each system, determine a state-space model. Show your choice of state variables on the original block diagram.

Hint 1: Look at Example 11.3.4.

Hint 2: Deal with blocks that contain 2nd-order transfer functions (e.g. in 11.9b) by breaking them up into two 1st-order transfer functions.

Hint 3: When deriving the state equations, use the fact that the following block diagrams are equivalent (you should prove to yourself that they are):



Problem 2: Properties of the State Transition Matrix

For the system

$$\dot{\vec{x}} = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} \vec{x}$$

1. Find the state transition matrix $\Phi(t)$.
2. Show that $\Phi(0) = I$.
3. Show that $\Phi^{-1}(t) = \Phi(-t)$.
4. Show that $\Phi(t_1 + t_2) = \Phi(t_2)\Phi(t_1)$.

Problem 3: Determining Stability and Solving the State Equations

Recall from lecture:

- The system $\dot{\vec{x}} = A\vec{x} + B\vec{u}$ is stable if all the eigenvalues of A lie in the left half-plane.
- Given initial conditions $\vec{x}(0)$ and input $\vec{u}(t)$ for $t > 0$, the state at any time t is given by:

$$\vec{x}(t) = \Phi(t)\vec{x}(0) + \int_0^t \Phi(t-\tau)B\vec{u}(\tau)d\tau \quad (1)$$

1. For the system

$$\dot{\vec{x}} = \begin{bmatrix} -1 & 1 \\ -1 & -3 \end{bmatrix} \vec{x}$$

- (a) Show whether this system is stable, marginally stable, or unstable.
 - (b) Find the transition matrix $\Phi(t)$ (you might need to consult your Laplace transform tables).
 - (c) Find the state response to the initial conditions $x_1(0) = 1$, $x_2(0) = -1$.
2. Do Problem 11.25 in Van de Vegte. (Note: the “output response” means $\vec{y}(t)$, not the state response $\vec{x}(t)$.)

Problem 4: Controllability

1. In one short paragraph, describe what it means to say that a system is uncontrollable.
2. Do Problem 11.35 in Van de Vegte.