16.06 Lecture 20
Effects of Proportional Control

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Today's Topics

1. Trade-off between stability and accuracy
2. First-order system
3. Second-order system
4. Third-order system
5. Angle and magnitude conditions

Reading: l.n

1 Control System Design

We have spent the first half of the semester learning how to analyze linear systems. Recall the four motivations for feedback:

1. Reduce the effects of parameter variations

2. Reduce the effects of disturbance inputs

3. Improve transient response characteristics

4. Reduce steady-state errors

We have learned how to quantify each of these effects for linear systems. We now understand the desirable and undesirable aspects of the transient and steady-state behavior of closed-loop systems. Once we have established performance specifications for a system, our primary concern will be the design of the compensator, i.e. how do we locate the closed-loop poles where we want them?

We have already seen one way to do this using state space methods. Now we will see a method of general system analysis in both the time domain and the frequency domain. Today we start by looking at the time domain. We will start by considering the effect of proportional control.

2 Trade-off between stability and accuracy

We have already seen the tradeoff between stability and accuracy. Today, we will investigate this issue in more detail. You will see that performance specifications cannot always be met by using proportional control. Often, we have to use a dynamic controller to get the response that we want. We will also see this in the lab when we try to design a controller for the Quansers.

- We have seen:
- Now, we will look at:
- Why are the closed-loop pole locations important?

We will consider the following closed-loop system (P-control):

We will consider three cases: $G(s)$ first, second and third order.

3 First-order system

$$G(s) = \frac{1}{s}$$

$$\frac{C(s)}{R(s)} =$$

pole at

- step response:
- stable for
- as K increases, system speed of response

4 Second-order system

$$G(s) = \frac{1}{(\tau_a s + 1)(\tau_b s + 1)}$$

$$\frac{C}{R} =$$

- closed-loop poles are at

- we see the trade-off mentioned earlier ...

- once the poles leave the real axis, the real part remains fixed at

- settling time cannot be reduced by increasing K. As K increases,

- You can also think about this in the context of state space and controllability. e.g. Consider the case $\tau_a = \tau_b = 1$. A state space model for the closed-loop system looks like the following:

5 Third-order system

$$G(s) = \frac{1}{(\tau s + 1)^3}$$

$$\frac{C}{R} = \frac{K}{\tau^3 s^3 + 3\tau^2 s^2 + 3\tau s + 1 + K}$$

C.E. :

We can get the roots by factorizing the C.E.:

With $K = 0$,

With $K = 1$,

With $K = 8$,

- Note how rapidly the damping ratio of the dominant pole pair deteriorates with increasing K
- With $K = 8$, closed-loop poles are on the imaginary axis ($\zeta = 0$). This value of gain is called K_{crit}.
- Any further increase in K will cause the complex pole pair to move into the RHP, leading to instability.

6 Higher-order systems

If the system is higher order, we can solve for the closed-loop roots using a graphical solution. For clues, we look at the three preceding sketches.

What do we see?

- For $K \to 0$:

- For $K \to +\infty$,

- number of branches =

- angle of branch =

- sum of angles

7 Angle and magnitude conditions

Can we approach this more formally?

Consider the following general system:

The loop gain function can be written in the general form

$$G_c(s)G(s)H(s) =$$

Recall that the typical factor $(s + a_i)$ is a vector from

The factor $(s + b_k)$ is a vector from

Recall that we can also express these vectors as

The closed-loop poles are the roots of the system C.E. $G_cGH + 1 = 0$, and so satisfy the equation

What is the RHS of this equation?

So the closed-loop poles are

$$\text{magnitude}(G_cGH) =$$

$$\text{phase}(G_cGH) =$$

The closed-loop poles therefore satisfy both of the following conditions:

1. **Angle condition**

2. **Magnitude condition**

Note that the A_i may be taken to be 1 if there are no open-loop zeroes.