16.06 Lecture 32

Gain and Phase Margins

November 20, 2003

Today's Topics:

1. Definitions of phase and gain margins
2. Interpretation of distance from –1
3. Examples

Commonly, in situations where there are no open loop poles in the RHP the criteria of gain and phase margins are used to evaluate the performance of feedback control systems. Typically the Nyquist diagram may look like this in the region near –1.

Now consider the vector from the –1 point to any point on the $G(j\omega)$ contour. It is in fact $G(j\omega)+1$. Thus the magnitude of the closed loop response at the frequency ω is always the ratio of these two vectors.

Clearly, if the contour passes through the –1 point then the ratio goes to infinity, at which point the system is at the threshold of instability, and it will have a resonant peak of infinite magnitude at that frequency.

Gain and phase margin are measures of the changes in magnitude and phase of the open loop system that would cause the contour to just pass through the –1 point. In particular they are defined as

Gain Margin-the magnitude of increased open loop gain that would cause the $G(j\omega)$ contour to pass through the –1 point.

Phase Margin-the amount of additional negative phase shift that would cause the $G(j\omega)$ contour to pass through the –1 point.

Consider the following diagram

The point C designates the place where $G(j\omega)$ crosses the negative real axis, so the phase angle of $G(j\omega)$ is –180 degrees at the point C. Gain margin in this diagram is the reciprocal of the distance from the origin O to the point C.

Increasing the gain by the gain margin (1/OC) will cause the contour to just pass through –1.

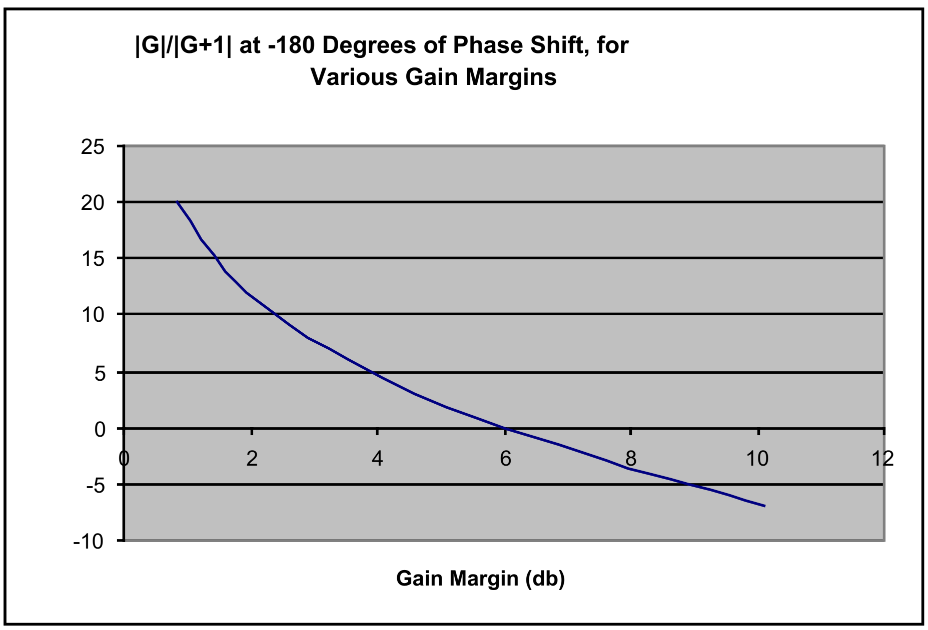
In similar fashion the angle ϕ in the diagram is the phase margin.

which is just the amount of additional negative phase shift that would cause the contour to pass through –1.

To get some idea about what values of gain margin give good performance we can evaluate the magnitude of the closed loop response, at –180 degrees of phase shift, as a function of gain margin. From the following diagram

we readily obtain the following equation for the magnitude of the closed loop response at the value of ω at which the phase of $G(j\omega)$ is –180 degrees.

The following diagram depicts this function



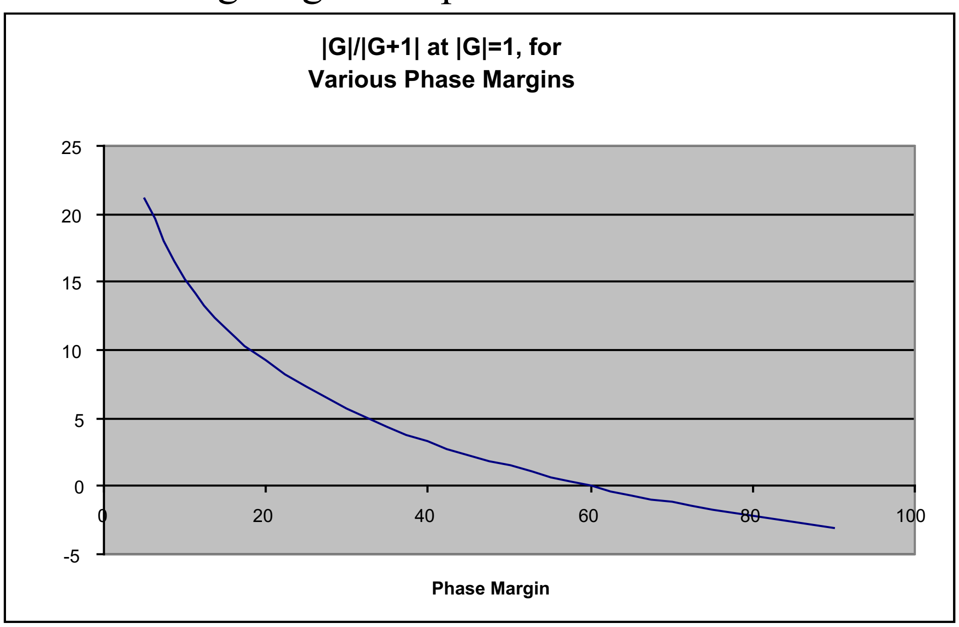
Note that below about 5 db of gain margin the closed loop response magnitude grows quickly, resulting in a resonance peak in the closed loop response. Hence it is desirable to have gain margins greater than about 5 db.

Now consider the following diagram.

The point B on the Nyquist contour is where the magnitude of $G(j\omega)$ just equals 1. Hence the triangle OAB is isosceles with its two equal legs both of unit length, so

Thus, at the frequency where the magnitude of $G(j\omega)$ is just equal to 1 the magnitude of the closed loop response is

The following diagram depicts this function



Note that below about 30 degrees of phase margin the closed loop response magnitude grows quickly, resulting in a resonance peak in the closed loop response. Hence it is desirable to have phase margins greater than about 30 degrees.

Example-

Recall the example that Prof. Willcox used to illustrate the design of a control system using a phase-lead compensator. The system transfer function was

And the closed loop system diagram was-

The lead compensator she designed was-

We wish to compare the gain and phase margins of the uncompensated and compensated systems. For the uncompensated system $G_c(s)$ will be a pure gain.

Both of these systems have an integrator in the forward path so they are both Type 1 systems. In order to do a fair comparison we will give both systems the same velocity constant. Thus we choose the gain for the uncompensated system as 29.8, so the uncompensated system will have the following open loop transfer function.

The following diagram illustrates the Nyquist diagrams for both systems near the –1 point.

As can be seen, both systems have infinite gain margins but the phase margin for the uncompensated system is only 21 degrees, while the phase margin for Prof. Willcox's design is 44 degrees. As result, the uncompensated system will have a resonant peak of about 9 db (M=2.8), while the compensated system will have a resonant peak of about 2.5 db (M=1.3).