16.06 Lecture 18

Controllability Continued

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Today's Topics

1. Controllability for systems with multiple inputs
2. Example

In situations when we have multiple inputs we obtain a similar result as for the scalar case. As before we want to be able to drive the state anywhere in its state space. Now the control is a multi-dimensional vector $\underline{u}(t)$. Controllability is achieved if $\underline{u}(t)$ can drive the integral.

anywhere in the state space in the time T. Similar to last time we now define vectors.

so the integral becomes

Let's look at the first term

which is a linear combination of other columns of the B matrix, similarly, for the second term

which is a linear combination of other columns of AB.

Extending this analysis to the other terms we conclude that the n dimensional vector represented by the integral

Is a linear combination of the columns of the matrices

Hence if the system is controllable there must be n columns of $B, AB, A^2B, \ldots$ which are linearly independent. Alternatively, controllability requires that the columns of $B, AB, A^2B, \ldots$ must span the state space.

It can be shown that only n of these matrices are necessary. Hence we have the general controllability as follows.

Controllability Condition- The system respresented by the matrices A, B is controllable if and only if the controllability matrix has n linearly independent columns

A not so obvious example-
Three masses on a frictionless surface

The system is at rest with

Differential equations describing this system are

We define the following states

Then, from the original differential equations we obtain

Now in order to make the arithmetic easy we choose

so

Our vector/matrix D.E.s become

Now we need to examine

where

Now

and similarly we will find that A^6B is a multiple of A^4B, A^7B is a multiple of A^5B etc. Hence we can not find 6 linearly independent columns of the controllability matrix. There are only four, so the system is uncontrollable.

One way to view this system is in terms of its oscillating modes. We can view the motions of the masses as oscillating in symmetric and asymmetric fashion. Both oscillate at identical frequencies. In the symmetric mode they move together in the same direction. In the asymmetric mode they move in opposite directions. The symmetric mode is controllable, the asymmetric is uncontrollable.

Now suppose we add a second control by imposing an additional force on one of the masses

The A matrix is unchanged but the B matrix is now

The six vectors underscored with $*$ are linearly independent so the system is controllable