MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.060: Principles of Automatic Control
Fall 2003

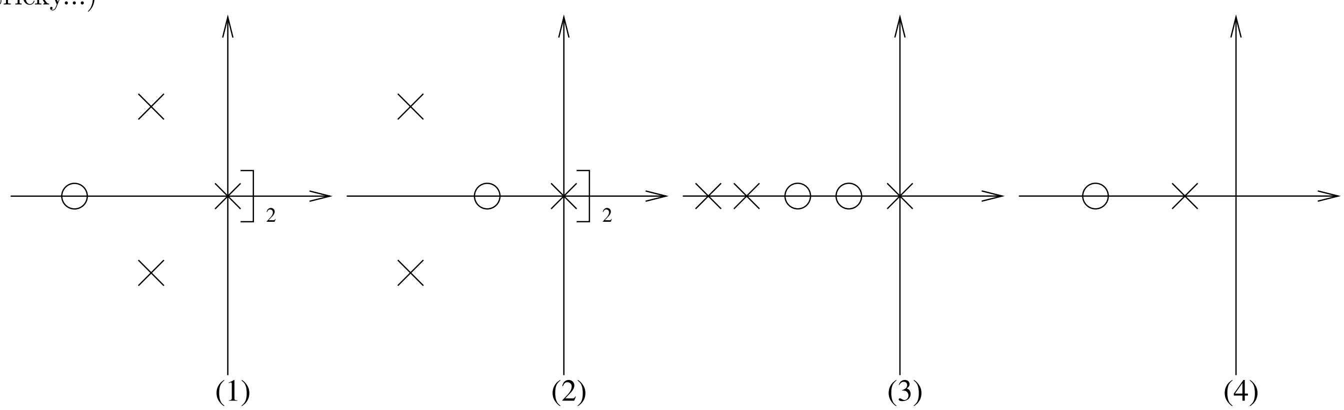
PROBLEM SET 7
Due: 11/6/2003

Important Reminder: Do Lab 2!

Complete your preliminary controller design and lab work by Friday, November 7. Your lab writeup will then be due the following week. This problem set is a bit shorter than normal so that you will have more time for the lab.

Problem 1: Root Locus Sketches for Positive and Negative Gain

Sketch (by hand) the root loci for the following pole-zero plots, for both $K > 0$ and $K < 0$. (The last one is tricky...)



Problem 2: Root Locus Sketches with Calculations

Sketch (by hand) the loci of the closed-loop poles for $K > 0$ for systems with the following open-loop transfer functions:

1. $G(s) = \frac{K}{(s+1)^2(s+2)}$
2. $G(s) = \frac{K(s+2)}{s(s+1)(s+3)(s+4)}$
3. $G(s) = \frac{K(s+3)}{s(s+2)(s^2+2s+2)}$
4. $G(s) = \frac{K(s+1)(s+3)}{(s+2)(s^2+4s+8)}$
5. $G(s) = \frac{K(s+2)^2}{s^2(s^2+2s+2)}$

For each, indicate:

- The number and direction of asymptotes.
- The point of intersection of the asymptotes with the real-axis.
- The angle of departure of any complex poles.

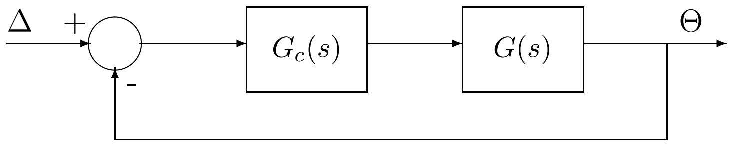
Also, for (1), (2), and (3): calculate K_{crit}.

Problem 3: PD Controller Design Using Root Locus

In lecture 9, we found that the linearized model for the attitude of a rocket (neglecting the moment of inertia of the engine) has a transfer function of the form:

$$G(s) = \frac{K}{s^2 - a^2}$$

In this problem, let $K = 1$ and $a = 1$. The open-loop system is clearly unstable, so we need to design a controller $G_c(s)$ to stabilize the system, as shown below. (Recall the input δ is the angle of the thrust vector, and the output θ is the rocket attitude angle.)



1. Suppose we use a proportional controller: $G_c(s) = K_c$. Sketch the root locus for the closed-loop system. Can we stabilize the rocket attitude by using this type of controller?

2. Now suppose we try to cancel the pole in the right half-plane, using a controller of the form $G_c(s) = \frac{K_c(s-1)}{s+p}$, where p is some positive real number. Sketch the root locus for $p = 2$. Is this type of controller likely to work in real-life? Why or why not?

3. Now use a PD controller of the form $G_c(s) = K_p + K_d s$. Using root locus methods, determine the values of K_p and K_d so that the closed-loop system will be stable with a 2% settling time of 4 seconds, and a damping ratio $\zeta \approx 0.7$.