MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.060: Principles of Automatic Control
Fall 2003

PROBLEM SET 9
Due: 12/04/2003

Problem 1: Representing a Pure Delay in the Frequency Domain

One situation in which frequency response techniques are useful is when a transfer function can not be represented easily in terms of poles and zeros. A very common example is a pure delay, which is always present in any control system implemented on a digital computer (e.g. Simulink). For a pure delay of time τ, the output $y(t)$ in terms of the input $u(t)$ is given by:

$$y(t) = u(t - \tau)$$

1. Using the definition of the Laplace transform, find the transfer function $G(s) = \frac{Y(s)}{U(s)}$ corresponding to a pure delay.
2. Draw the polar plot for $G(s)$ for a time delay of 1 second. (The polar plot is the part of the Nyquist plot corresponding to $0 < \omega < \infty$.)

Problem 2: Nyquist Plots with Negative Gain

For each of the following open-loop transfer functions, draw one Nyquist plot for $K > 0$, and then another Nyquist plot for $K < 0$. In each case, use the Nyquist stability criterion to determine the range of values of the gain K for which the closed-loop system is stable. Then draw the root locus plot (for positive and negative gain) to confirm your results.

1. $G(s) = \frac{K}{(s+1)(s+2)}$
2. $G(s) = \frac{K}{s(s+2)}$
3. $G(s) = \frac{K(s+1)}{s+2}$
4. $G(s) = \frac{K}{s^2(s-2)}$

Problem 3: Gain Margin, Phase Margin, and the Nichols Chart

Recall that gain margin and phase margin give a quantitative measure of the relative stability of the closed-loop system. The gain margin is the maximum factor by which you could multiply the gain of the open-loop transfer function $G(s)$ before the closed-loop system $\frac{G}{1+G}$ becomes unstable. The phase margin is the maximum amount by which you could decrease the phase (angle) of $G(s)$ before the closed-loop system becomes unstable.

For real-life systems, $G(s)$ is never a perfectly accurate model of the system (think of the Quansers!). But if the gain and phase margins are high, then the closed-loop system will still be stable even if the real transfer function turns out to be slightly different from $G(s)$. On the other hand, if the gain and phase margins are low, then a relatively small difference between $G(s)$ and the real system could mean that the closed-loop system is unstable. Therefore, it is very desirable to have high gain and phase margins.

For the open-loop transfer function $G(s) = \frac{3}{s(s+1)(s+2)}$:

1. Draw the Nyquist plot and the root locus plot. On the Nyquist plot, indicate how the gain margin and phase margin are defined.

2. Is the closed-loop system stable? Calculate the gain margin and the phase margin for this system. (You can use Matlab or some other tool for the calculations.)

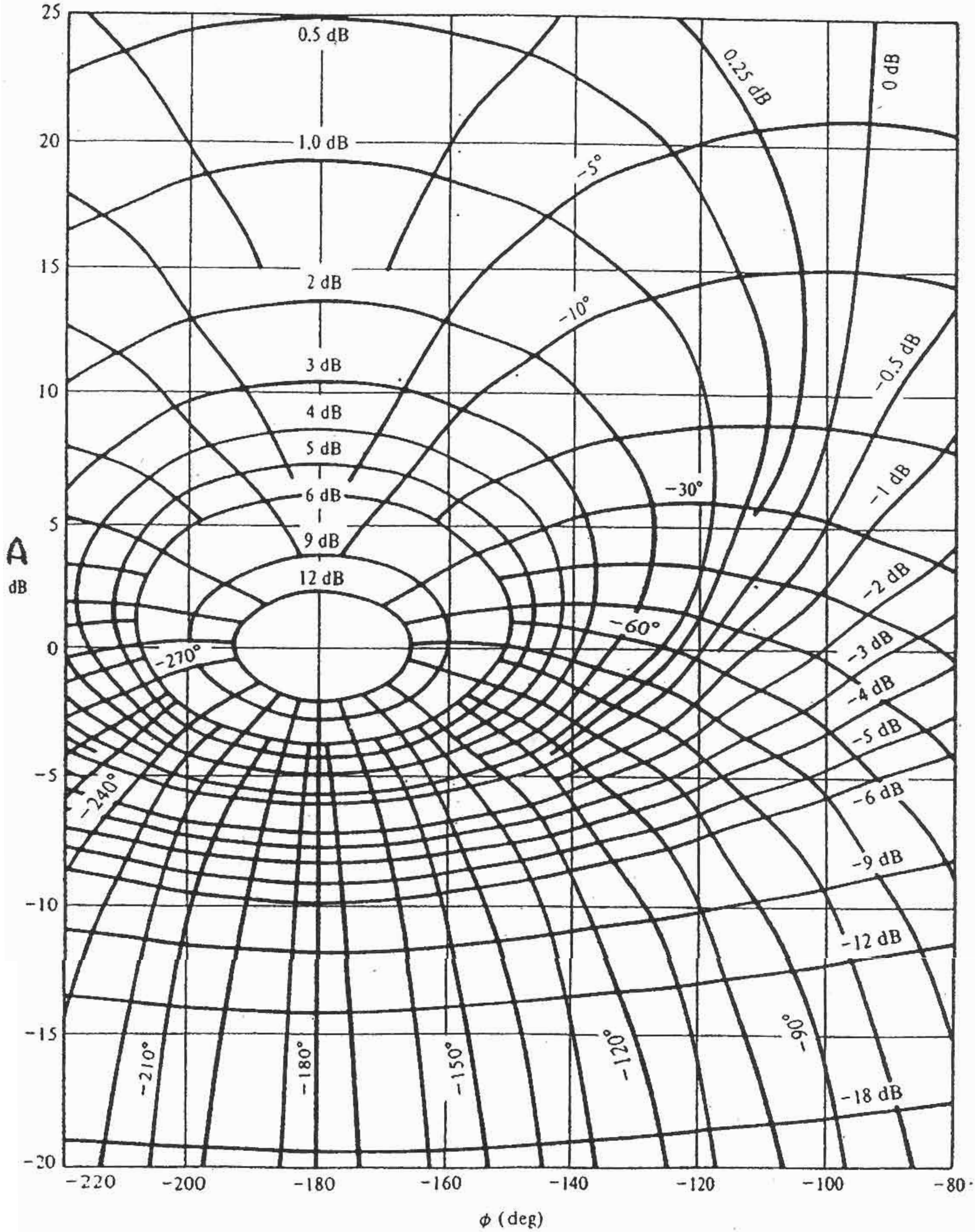
3. Calculate K_{crit} for this system. How is K_{crit} related to the root locus gain of G and the gain margin?

   Recall that a Nichols chart is a plot of magnitude vs. phase for the open-loop transfer function $G(j\omega)$. The contours on the Nichols chart show the magnitude and phase of the closed-loop transfer function $\frac{G}{1+G}$. So for a given magnitude and phase of G at a given frequency ω, the Nichols chart shows the corresponding magnitude and phase of $\frac{G}{1+G}$ at the same frequency.

4. Using the same data as for the Nyquist plot, sketch $G(j\omega)$ on the attached Nichols chart. On the same chart, indicate the gain margin and the phase margin. (Hint: The point ($-180°$, 0 dB) corresponds to the -1 point on the Nyquist plot.)

5. Using the Nichols chart, obtain an approximation for the maximum amplitude (or magnitude) of the closed-loop transfer function, and the frequency at which it occurs.

6. Looking again at the Nichols chart, if the gain and phase margins were higher, how do you think that would affect the peak amplitude of the closed-loop transfer function?



0.5 dB
1.0 dB
2 dB
3 dB
4 dB
5 dB
6 dB
9 dB
12 dB
0.25 dB
0 dB
−0.5 dB
−1 dB
−2 dB
−3 dB
−4 dB
−5 dB
−6 dB
−9 dB
−12 dB
−18 dB
−5°
−10°
−30°
−60°
−90°
−120°
−150°
−180°
−210°
−240°
−270°
A
dB
φ (deg)
25
20
15
10
5
0
−5
−10
−15
−20
−220
−200
−180
−160
−140
−120
−100
−80