

# 16.06 Lecture 22

## Root Locus Examples

Karen Willcox

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### Today's Topics

1. Angles of departure
2. Negative gain
3.  $K_{crit}$
4. Root locus examples

**Reading:** 6.4

# 1 Rule 10

(Continued from L21.)

Angles of departure from open-loop poles and angles of arrival at open-loop zeroes are important because aero/astro toys can have complex conjugate poles and zeroes near the  $j\omega$  axis. We determine these angles by applying the angle condition to a trial point very close to the pole or zero.

Example 1:

Example 2:

## 2 Negative gain

### 2.1 First-order system

C.E. is:

### 2.2 Second-order system

C.E. is:

Roots:

$s_1$  and  $s_2$  are

## 2.3 Third-order system

C.E. is:

## 2.4 General $K < 0$ case

Note that for negative gains, the angles must sum to

The entire real axis is

### 3 $K_{crit}$

At what critical value of  $K$  in the third-order case are the roots on the  $j\omega$  axis?

At this point,  $s = j\omega$  and the C.E. is

We can solve the above equation for  $\omega$  and  $K_{crit}$  by setting the real and imaginary parts to zero:

## 4 Root Locus Examples

4.1  $G_c G(s) = \frac{K(s+3)}{(s+1)(s+2)}$

4.2  $G_c G(s) = \frac{K(s+3)}{(s+1)(s+2)(s+4)}$

### 4.3 Near view/far view

$$\begin{aligned} G(s) &= \frac{K(s+1)}{s^2(0.5s+1)(0.1s+1)} \\ &= \frac{k(s+1)}{s^2(s+2)(s+10)} \end{aligned}$$

where  $k = 20K$ .

(a) Assume  $|s| \ll 10$

$$G(s) \approx$$

(b) Assume  $|s| \approx 10$

$$G(s) \approx$$

The combination of (a) and (b) yields: