MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.060: Principles of Automatic Control
Fall 2003

PROBLEM SET 8
Due: 11/26/2003

Note: This problem set is due at the beginning of class on **Wednesday**!

Problem 1

The input to a system with transfer function $G(s)$ is $r(t) = A\sin\omega t$. Show, by partial fraction expansion and complex number manipulation, that the output in steady-state is:

$$c_{ss}(t) = AM(\omega)\sin(\omega t + \phi(\omega))$$

where $M(\omega) = |G(j\omega)|$ and $\phi(\omega) = \angle G(j\omega)$.

Problem 2: Nyquist Plots

For each of the following open-loop transfer functions, draw a plot of the s-plane showing the poles and zeros of $G(s)$ and the Nyquist D-contour. Then sketch (by hand) the Nyquist plot for each system. Label points corresponding to important frequencies (e.g. $\omega = 0, \pm\infty$) and use arrows to show the direction of increasing frequency.

1. $G(s) = \frac{K}{s}$
2. $G(s) = \frac{K}{s^2}$
3. $G(s) = \frac{K}{s^3}$
4. $G(s) = \frac{K}{s+1}$
5. $G(s) = \frac{K(s+2)}{s+1}$
6. $G(s) = \frac{K(s+1)}{s+2}$
7. $G(s) = \frac{K}{(s+1)(s-1)}$

Problem 3: Using Nyquist Plots to Determine Stability

1. Sketch (by hand) the Nyquist plot and the root locus plot (for $K > 0$) for each of the following open-loop transfer functions. Use the Nyquist stability criterion ($N = Z - P$) to determine the range of values of the gain K for which the closed-loop system is stable.

   Hint: Convert each transfer function to root locus form.

   (a) $G(s) = \frac{K}{(s+1)(\frac{s}{10}+1)}$

   (b) $G(s) = \frac{K}{(s-1)(\frac{s}{10}+1)}$

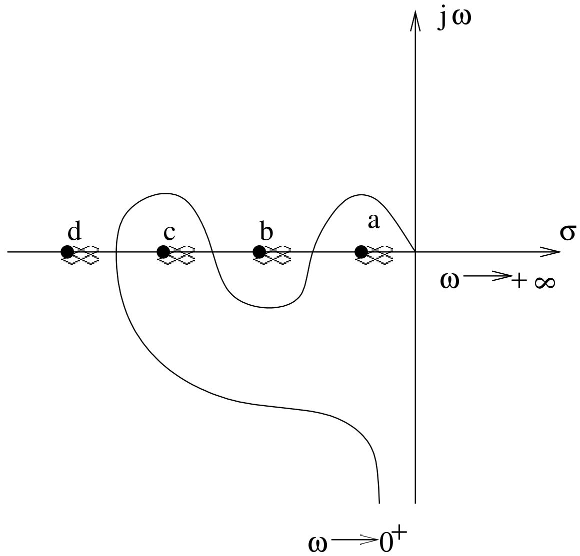
   (c) $G(s) = \frac{K}{s(s+1)}$

   (d) $G(s) = \frac{K}{s(s+1)(\frac{s}{10}+1)}$

(e) $G(s) = \frac{K}{s^2(s+1)}$

(f) $G(s) = \frac{K}{s^2(s-1)}$

2. This figure shows the polar plot for an open-loop transfer function with a single pole at the origin, plus other poles and zeros in the left half-plane. Complete the Nyquist diagram, and determine the stability of the closed-loop system for each of the four cases when the -1 point is at (a), (b), (c), and (d). Is the closed-loop system stable for very small values of postive gain? What about for very large values of positive gain?



3. Now suppose the open-loop transfer function in (2) has three poles at the origin, instead of one. What does the Nyquist plot look like now? For what values of positive gain is the closed-loop system stable?