

Practice State-Space Problem for Quiz 1

16.060: Principles of Automatic Control

October 8, 2003

For the following differential equation:

$$\ddot{w} + 5\dot{w} + 6w = \ddot{r} + \dot{r} + 2r$$

1. Derive a state-space model in the form:

$$\begin{aligned}\dot{\vec{x}} &= A\vec{x} + B\vec{u} \\ \vec{y} &= C\vec{x} + D\vec{u}\end{aligned}$$

Take r to be the input and w to be the output.

2. Draw a block diagram for the system, and clearly label the input, the state variables, and the output. (Hint: look at the diagram on page 4 of the Lecture 13 notes.)
3. Using your state-space model, determine the transfer function $G(s)$ from the input to the output.
4. Calculate the state-transition matrix $\Phi(t)$.
5. If the initial state is $\vec{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and there are no inputs, what is the state vector $\vec{x}(t)$ at time $t > 0$?