

16.06 Lecture 15

Solutions of State Space Differential Equations

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October 8, 2003

Today's Topics

1. General solution of state space D.E.s
2. Quanser example solution for constant input.
3. Stability

Complete Solution of the State D.E.s

Thus far we have obtained the homogeneous solution for the state as

But we need the total solution to

for non-zero inputs. We assume a solution of the form

where $\underline{f}(t)$ is some, as yet undetermined, vector valued function of time.

First we differentiate

and substitute our assumed solution

Referring back to our original differential equation it is apparent that we must have

or, using Property 1 of state transition matrices

Now integrate both sides from time zero to time t

so

Referring back to the assumed solution we know that

so

and hence

so, using Property 2 of state transition matrices

The integral on the right is the vector/matrix equivalent of the scalar convolution integral. In particular the term

transitions the incremental effect of the input from the past time τ to the current time t

The integral serves to sum up all such increments from the initial time to the current time.

Example

We will use this solution to obtain the response of the Quanser to a step input in motor voltage

The state matrices are

The initial state and input are

we know that the state transition matrix is

SO

These are the equations of an ellipse in the x_1, x_2 plane. In particular we know that

so

This is the trajectory in state space for no damping. if there is positive damping the solution spirals in. If there is a negative damping the solution spirals out

System Stability System stability is determined by the behavior of the homogeneous or unforced response of a system, which is the solution of

In particular, we know that this solution is determined by the initial condition on \underline{x} and the state transition matrix

Thus, system stability is determined by the behavior of $\Phi(t)$. Now we know that

and

So the denominator of each term is the determinant of $sI - A$.

Also, the system characteristic equation is

so every element of the L.T. of $\Phi(t)$ has a denominator with the same roots as the characteristic equation. Hence we have the rather obvious results-

System Stability

The system

is stable if and only if the roots of

all lie in the left half of the s plane

My model for the Quanser is not stable because its roots are on the imaginary axis