# 16.06 Lecture 11

# State Space

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**September 29, 2003**

## Today's Topics

1. The concept of system state
2. State vector definition
3. State space representation of LTI systems

The state space approach models systems using linear vector space methods.

The concept of a state vector (array) is important because it completely represents the current status (state) of a system.

Example: A Capacitor

Typically, there are many possible ways to define the state. Charge $q$ or voltage $v_c$ can be states. If $v_c$ is chosen as the state then the constituitive relation for the capacitor characterizes the behavior of the state

If we have the simple circuit system

We have two constitutive relations

so the equation for the system is

with the solution

and $v(t)$ is the system state.

Furthermore, if we know the state at any particular time $t_1$, then we know the state for all future time.

If there are no inputs (homogeneous system) then the state is sufficient to predict the entire future behavior of the system.

Definition of the state vector for a system.

We define the state vector as a $\underline{x}(t)$

In general the system will also have inputs $\underline{u}(t)$ and outputs $\underline{y}(t)$

As we have said a number of times, in this course we will only study linear, time invariant systems (LTI). Any LTI system can be characterized with four constant matrices- A,B,C and D, as follows

A block diagram representation is

where

Simple example of a second order system

The externally applied force $f$ is the input and the output is the displacement $d$ of the mass away from its rest position ($d = 0$)

differential equation for this systems is, using $(F = ma)$

This is a second order system so we need two states. Choose

We need the time derivatives of these two states

Thus we can write the vector/matrix equations

Also, any non-singular transformation of the state of a system is also a state of the system. For example, suppose the $(n \times n)$ matrix $T$ is a non-singular transformation of the state $\underline{x}$ to a new state $\underline{x}'$.

Then

so

If we then define

the equations for the new state are

Similarly

so

and hence

so the input and output are the same as before but we now have a transformed state $\underline{x}'$.