16.06 Lecture 14

Quanser Model and State Transition Matrices

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Today's Topics

1. State space model of the Quanser
2. Homogeneous solution of state D.E.s
3. State Transition matrices

Quanser State Space Model

We will now model the Quanser in state space and we will use it as the example in many of our remaining developments. You developed the following transfer function to represent the Quanser

where the output is the angle of the arm with respect to its nominal zero location and the input is voltage to the motor. Recall that damping is very small and here it is assumed to be zero.

Define two states

Also, from the transfer function we obtain a D.E.

or

Hence our state D.E. is

Also, we assume we have two outputs which are the angle and angular velocity

The system block diagram is

Let's determine the matrix transfer function

From last time

Now

and

so

General Time Domain Solutions

We will now develop the general time domain solutions to the state space equations. We have the vector/matrix D.E.

First consider the unforced (homogeneous) solution. It satisfies the equation

Recall, That in the scalar case this equation will be

with the exponential solution

and

With this solution in mind we define a matrix exponential as where $\Phi(t)$ is called the State Transition Matrix. We then assume the solution

cleary we have identity at time zero. Also, if we differentiate we get

And if we substitute the original assumed solution into this equation we get

Now the transition matrix is very important. When there is no input it determines how the system "transitions" from to state as time evolves

It is the general homogeneous solution

Now $\Phi(t)$ has some rather obvious but important properties which are proven in Vande Veght

Property 1

Property 2

One good way to obtain state transition matrices is by Laplace Transforms. We know that

Now define the Laplace Transform of Φ as a matrix of L.T.'s of each element of Φ

Then, from the D.E.

or

so

and this

or, finally

Let's find the state transition matrix for the Quanser

Example

Earlier we found that for the Quanser

hence

Now suppose we have the initial conditions

so

and thus

Now these equations can be combined to yield

which is the equation of an ellipse. Hence the state space trajectory looks like