MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.060: Principles of Automatic Control
Fall 2003

Quiz 2 Practice Problems

State-Space

1. For the state model:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; \quad \mathbf{y} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x}$$

   (a) What are the poles of the system? Is the system stable?
   (b) Calculate the transfer function $G(s)$
   (c) Calculate the state transition matrix $\Phi(t)$
   (d) Use the convolution integral to find the output response of the system to a unit step input and the initial conditions $\mathbf{x_o} = [1\ 1]'$.

2. Check the controllability of the following system:

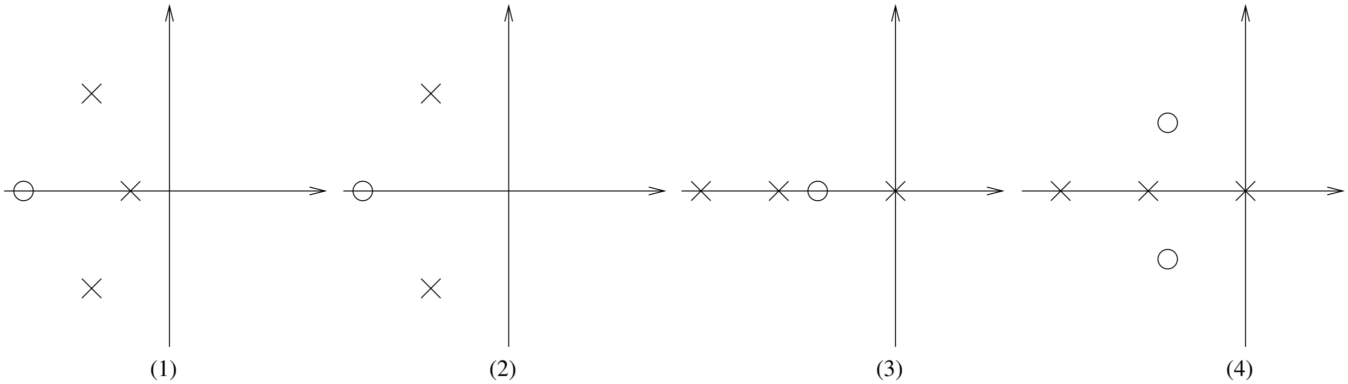
$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

   Does the answer depend on the values of a, b, and c?

3. Full state feedback controller design: Problem 12.9 in Van de Vegte.

Root Locus

1. Sketch root locus patterns for the following, for positive and negative gain:



2. Sketch by hand the loci of the closed loop system poles for varying $K > 0$ for systems with the following open loop transfer functions. Indicate the number, direction, and point of intersection of the asymptotes, the angle of departure of the loci from complex poles, and $K_{critical}$.

   (a) $G(s) = \frac{K}{s(s+2)(s+5)(s^2+4s+13)}$

   (b) $G(s) = \frac{K(s+5)}{s(s+2)(s^2+2s+5)}$

3. The elevation dynamics of the Quanser include a set of complex conjugate poles with $\omega_n = 0.7$ rad/s, and $\zeta = 0.08$, and first-order lag with a time constant of 1/6 seconds. The standard gain of the plant is 0.4.

   Design a PD controller so that the closed-loop system has an undamped natural frequency of 2.5 rad/s and a damping ratio of 0.5.