

16.06 Lecture 12

State Space Modeling

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Today's Topics

1. State space model for nth order D.E.s
2. State space models for transfer functions
3. Examples

We want a method for creating an n dimensional state space model from any set of LTI ordinary differential equations of order n . Suppose an output w (scalar) is related to in input r (scalar) by the LTI ordinary D.E.

We define states as w and it's $n - 1$ derivatives

So we have n states (same as the order of the D.E.). Now we differentiate x , through $x_{(n-1)}$

We still need the derivative of x_n . It is

This one we obtain from the original D.E.

Then we substitute from our state definitions

We thus have n first order differential equations in terms of states and the input. In vector matrix notation.

Also, since our output is w

Since we know A, B, C and D we have a state space model.

We can draw a block diagram of this system as follows

Example-

Then there are two states(second order equation)

So, from the original D.E. we have

and the vector/matrix equations become

and $w = x_1$ is the output so

Now suppose, instead of a D.E. we have a transfer function with no zeros. So

This is just the transfer function of our original D.E. so we immediately have the state space model for this transfer function. It's the one derived above!

But what if there is a zero? Then, for example

Break this down into two transfer functions, one with only poles the other with only zeros

The first block can simply be represented as the state space model we developed earlier. However, now the output is a linear combination of derivatives of x_1 . In particular

So, in the time domain we have

or finally

Hence the zeros are created by the elements of the C matrix.

Example

Break $G(s)$ down into two T.F.'s

Obtain the state space model for the first block. Its D.E. is

Define states as

Then from the D.E.

So our state D.E. is

For the second block we have

or in the time domain

so

Let's look at this system in terms of our block diagram

The zero is created by feeding the x_2 state forward. The feed forward combination is created by the C matrix.