MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.060: Principles of Automatic Control
Fall 2003

PROBLEM SET 1
Due: Thursday 9/11/2003

Reminder: The MATLAB/Simulink tutorials will take place on Monday (9/8) at 7pm and 8pm, and Tuesday (9/9) at 7pm Each session will last about one hour.

Problem 1: Why Feedback

Pick an example of a closed-loop system that you are familiar with (e.g. temperature control system in an office, cruise control in a car). Draw a simple block diagram of that system, similar to the one shown in Figure 1. For the system you choose, specify the physical input, the physical output, and a possible source of disturbance and noise. Also write down the physical devices that constitute the "controller", the "plant", and the "sensor", but don't worry about writing any transfer functions! Discuss briefly (in one paragraph) how effective or ineffective the control system would be without feedback.

Problem 2: Block Diagrams and Concept of Gain

The following block diagram represents a generic closed-loop linear system consisting of a reference input $R(s)$, a disturbance $D(s)$, and sensor noise $N(s)$ and output $C(s)$.

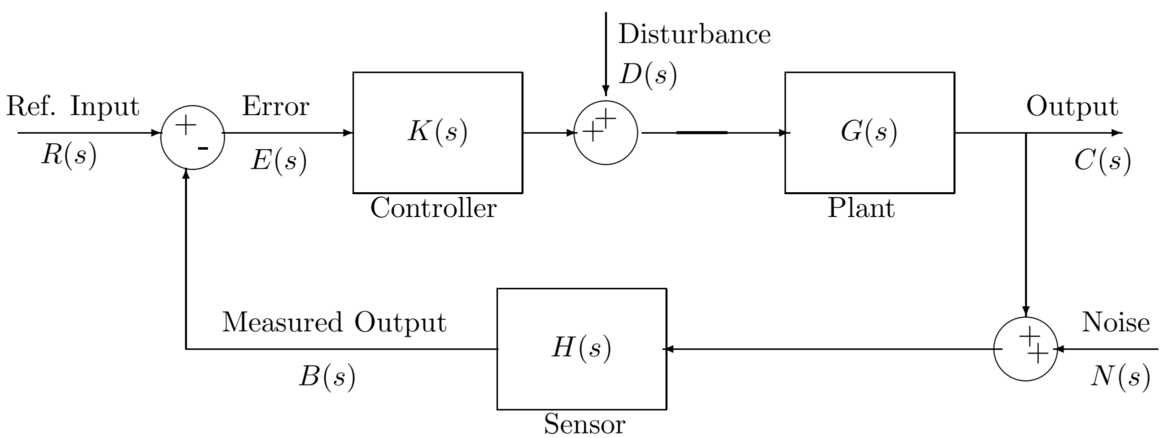

Figure 1: Feedback Loop

1. Evaluate the transfer functions relating the output $C(s)$ to each of the inputs $R(s)$, $D(s)$, and $N(s)$. *Hint: since the system is linear use the property of superposition. For example, to calculate the transfer function $C(s)/R(s)$, set $D(s)$ and $N(s)$ to zero.*

2. Assume that $H(s) = 1$ (i.e. unity feedback) and $K(s) = K$ (i.e. the controller is a simple amplifier). Assume also that there is no noise or disturbance (i.e. $N(s) = 0$ and $D(s) = 0$). Derive a simple expression for the error $E(s) = R(S) - B(s)$, in terms of $R(s)$, $G(s)$, and K.

3. Consider the following plants: $G_1 = \frac{1}{s^2+2}$ $G_2 = \frac{1}{s(s+2)}$ and take $R(s)$ as a unit step ($R(s) = \frac{1}{s}$). Again, assume $H(s) = 1$, $K(s) = K$, $N(s) = 0$, and $D(s) = 0$. For each of the two plants, use the Final Value Theorem (p.11) to find the steady state value (i.e. as t goes to infinity) of the error.

4. In general, you want the steady-state measured output to match the reference input. The difference is called the "steady-state error", and one of the objectives of feedback is to minimize this quantity. In Problem 2, Part 3, what happens to the steady-state error as you vary the gain K? Is your answer different for the two plants?

Problem 3: Math Questions

3.1: Complex numbers

For each of the following indicate real and imaginary parts, and the modulus and the argument.

1. $z_1 = e^{-\pi j}$
2. $z_2 = 2e^{-\frac{\pi}{3}j}$
3. $z_3 = \frac{2+j}{1+3j}$
4. $z_4 = \frac{1}{(1+j)^2}$

3.2: Computing Laplace transforms

Define $u(t)$ to be the unit step function: $u(t) = 1$ if $t > 0$, $u(t) = 0$ otherwise.

Using the definition of the Laplace transform:

$$\mathcal{L}[f(t)] := \int_0^\infty f(t)e^{-st}dt$$

evalulate Laplace transforms of the following functions:

1. $f_1(t) = u(t)$ (the step function itself)
2. $f_2(t) = tu(t)$ (a ramp function)
3. $f_3(t) = e^{-t}u(t)$ (exponential decay)

Remember that taking the integral of a function is equivalent to multiplying its transform by $1/s$.

3.3: Computing inverse Laplace transforms

Compute the inverse Laplace transforms using partial fractions expansion and the table on page 12:

1. $F_1(s) = \frac{3}{s^2+2s-3}$ $\quad \Re e\{s\} > -1$
2. $F_2(s) = \frac{1}{(s^2+4)}$ $\quad \Re e\{s\} > 0$
3. $F_3(s) = \frac{s^2+s+2}{(s+1)^3}$ $\quad \Re e\{s\} > -1$

3.4: Solving differential equations using Laplace transforms

Solve the following differential equations using Laplace transforms (again, $u(t)$ is the unit step function):

1. $2\dot{y}(t) + 3y(t) = e^{-t}$, with $y(0) = 1$;
2. $\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = u(t)$, with $\dot{y}(0) = 1/2$ and $y(0) = -1$