MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.060: Principles of Automatic Control
Fall 2003

PROBLEM SET 6
Due: 10/30/2003

Problem 1: Controllability for Systems with Multiple Inputs

Determine whether the following systems are controllable:

1.
$$\dot{\vec{x}} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \vec{u}$$

2.
$$\dot{\vec{x}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \vec{u}$$

Problem 2: Pole Assignment with Full State Feedback

1. For the following system:

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -23 & -9 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u$$

   (a) Where are the poles of the open-loop system?

   (b) Design a full-state feedback controller so that the poles of the closed-loop system are at $s = -3$ and $s = -4 \pm 4j$. (The input to the closed-loop system should be in the form $u = u_c - K\vec{x}$.)

2. In Lecture 9, we looked at the problem of attitude control for a rocket. Neglecting the moment of inertia of the engine, the linearized equation for the rocket attitude is:

$$I\ddot{\theta} = l_1 c_n \theta + l_2 T \delta$$

   where I is the moment of inertia of the rocket, θ is the angle of the rocket attitude, δ is the angle of the thrust vector, c_n is the aerodynamic force coefficient, T is the thrust of the rocket engine, and l_1 and l_2 are the distances from the center of mass of the rocket to the bottom and top of the rocket respectively. For this problem, use the following values: $l_1 = l_2 = 1$, $I = 1$, $T = 4$, $c_n = 4$.

   (a) Derive a state space model for this problem. Take the input to be δ, the output to be θ, and the state variables to be $x_1 = \theta$ and $x_2 = \dot{\theta}$.

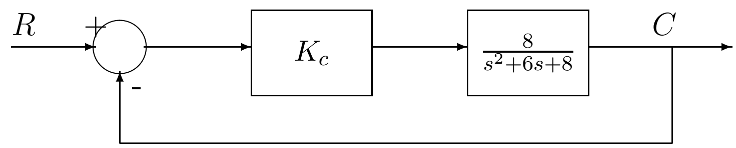
   (b) Where are the open-loop poles of this system? Would you recommend operating the rocket without a closed-loop control system?

   (c) Assume that we have sensors that can accurately measure θ and $\dot{\theta}$. Design a full-state feedback controller so that the closed-loop system has poles at $s = -2 \pm 2j$.

   (d) What limits where we can place the closed-loop poles of the system? (Think about what the input would look like if we designed a controller that assigns the closed-loop poles to be at $s = -1000 \pm 1000j$.)

Problem 3: Proportional Control

For the following system with proportional controller K_c:



1. Draw the root locus plot.
2. Design a proportional controller so that the closed-loop system is critically damped.
3. Design a proportional controller so that the closed-loop system has a damping ratio of $\frac{1}{\sqrt{2}}$ (≈ 0.7) and a 2% settling time of $\frac{4}{3}$ seconds.
4. Suppose we allow the gain of the proportional controller to be negative. What value of the controller gain will cause the closed-loop system to go unstable?

Problem 4: Root Locus

Sketch (by hand) the root locus plots for each of the following systems. In (4), also calculate the angle of departure of the loci from the complex poles.