

16.06 Lecture 4

Disturbances and Sensitivity

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Today's Topics

1. Sensitivity to parameter variation
2. Static and dynamic sensitivity
3. Effects of external disturbances

Reading: 4.1,4.2

1 Recall the motivations for feedback!

1. Reduce the effects of parameter variations
2. Reduce the effects of disturbance inputs
3. Improve transient response characteristics
4. Reduce steady-state errors

2 Disturbances and parameter variations in control systems

Recall the standard block diagram we saw in Lecture 2:

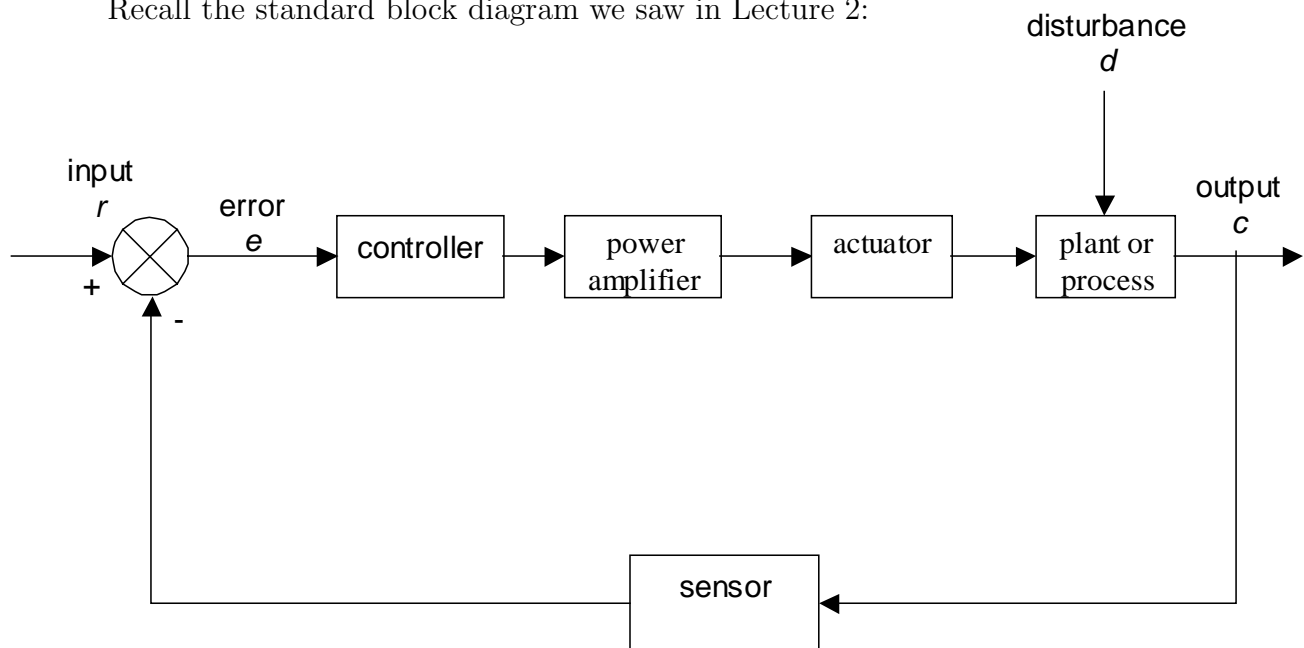


Figure 1: Standard block diagram.

- Disturbances
- Parameter variations
- Examples:

3 Sensitivity to parameter variation (vdv 4.2)

- Consider the following system:

- The system transfer function is:

Note that we use $G(s)$ to describe the open-loop transfer function of one particular component of a system, and $T(s)$ to describe the transfer function of the entire closed-loop system.

- If the loop gain function $G_cGH \gg 1$, then

and

- Also, the sensitivity of closed-loop performance to variations in G_cG is

- Think about a simple system where the blocks are gains (v.d.v. Ex. 4.2.1):

- What would happen if we used open-loop control? Now $\frac{C}{R} =$

- So, let's add unity feedback. Now $\frac{C}{R} =$

4 Example: Operational Amplifier

An op amp is a very important general purpose device. It is the basic element of an analog computer, and can also be used as a summing junction or to realize dynamic compensators (integrator, differentiator, phase lead/lag). The amplifier has a very high gain and a very high input impedance. See vdv 3.6 for more details if you are interested.

- In manufacturing, the value of A can vary from 300,000 to 1,000,000.

We set $A = 300,000$ and $H = 0.1$ as nominal values.

- The transfer function is

- We want to find the percentage change $\partial T/T$ in response to the percentage changes $\partial A/A$ and $\partial H/H$. We define formally the sensitivity

function S as follows:

- Now, for the op amp example:

In words, this result means:

- Using the numbers above, we find that

- We can also calculate the sensitivity to variations in H :

In words, this result means:

- Using the numbers above, we find that
- This is an example which shows the sensitivity of the TF to parameter variations. Sensitivity to model uncertainty is also an important issue, but you will not see that until 16.30.

5 Dynamic Sensitivity

The op amp example did not have dynamics. What happens when we include them?

e.g.: Consider a motor position servo

$$T =$$

Repeating the algebra in Section 4, we get:

$$S_K =$$

S_K is plotted as a function of frequency. But for now, we will not do this.

Therefore, we define....

6 Static sensitivity function

For the motor position servo S_K , we set $s = 0$.

Why set $s = 0$? Remember that steady-state corresponds to $t \rightarrow \infty$ in the time domain, or $s \rightarrow 0$ in the frequency domain.

So when we substitute $s = 0$ in the above expression for S_K , we get

$$S_{K_s} =$$

What does this mean?

7 Effects of external disturbances

We can consider the effects of R and D separately, because the system is linear and superposition applies. i.e. to get the transfer function from R to C , we set $D = 0$ and compute $\frac{C}{R}$; to get the transfer function from D to C , we set $R = 0$ and compute $\frac{C}{D}$.

What are the desirable transfer functions we would like our system to have?

The transfer function from disturbance to output is given by:

If $G_c G_1 G_2 H \gg 1$, then

If we want $\frac{C}{D}$ to be small, then

8 What about performance?

We have just seen how to make $\frac{C}{D}$ small, but what about good response to R ?

It is not much good if our closed-loop system has good disturbance response but does not produce the desired output! The question is, can we achieve both good disturbance rejection and good response to R at the same time?

The transfer function between R and C is given by

with $H = 1$ (we know how to do this) and $G_c G_1 G_2 \gg 1$, then

”good tracking”

We can get both good ”tracking” and good disturbance rejection

if