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16.060 QUIZ
October 24, 2002
All questions are worth 10 points each.

1. Answer TRUE or FALSE (you may include a sentence of explanation if you wish)

a. A linear system with feedback may be characterized as stable for certain inputs and as unstable for other inputs.

FALSE: stability is a property of the system, not the inputs

b. Increasing the root-locus gain of a first-order, linear feedback system always tends to make the system more stable.

(wait till we study root locus methods)

c. Good tracking and good disturbance rejection are typically the goal of control system design, but cannot be achieved simultaneously with the use of feedback control.

FALSE: we can get both by putting high gain between where R and D enter the loop.

d. The open-loop zeroes of a linear system can be moved with feedback.

FALSE: can only move OL poles with feedback

e. The root-locus gain is important because it determines steady-state errors.

FALSE: use standard gain to determine e_{ss}

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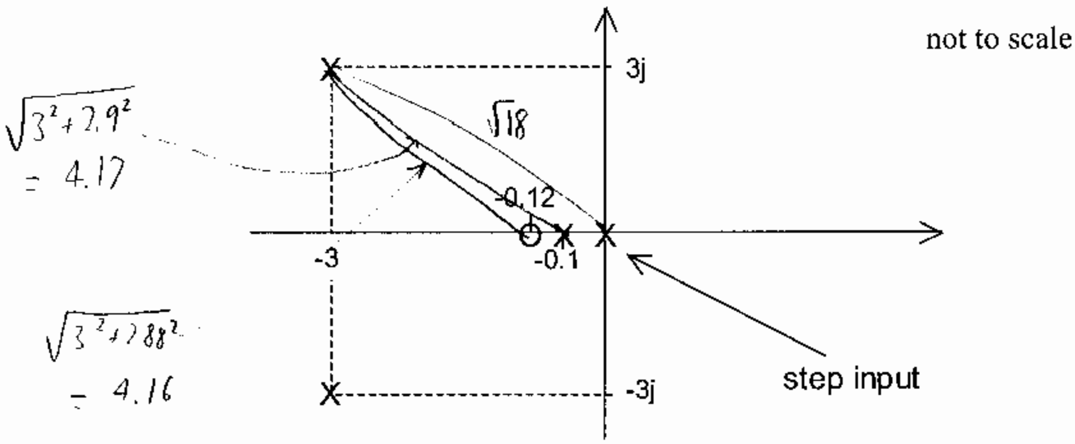
2. Consider the system represented by the following pole/zero diagram, where the pole at the origin represents a step input. The system has unity gain.



a. Write down the form of the step response *c(t)*. No numbers are necessary for the residue magnitudes and angles, but define your notation.

b. What is the root locus gain of the system?

c. Calculate the magnitude of each of the residues.

d. What is the decay time of each exponential mode?

e. Is there a dominant mode or modes? Explain the rationale for your answer in a few words.

(a) $c(t) = K_1 + K_2 e^{-0.1t} + 2K_3 e^{-3t} \cos(3t + \theta)$

(b) System has unity standard gain

$$\Rightarrow C(s) = \frac{s+0.12}{0.12} \cdot \frac{1}{s} \cdot \frac{0.1}{s+0.1} \cdot \frac{18}{s^2+6s+18} = 15 \frac{s+0.12}{s(s+0.1)(s^2+6s+18)}$$

$\Rightarrow K_{RL} = 15$

(c) $C(s) = \frac{K_1}{s} + \frac{K_2}{s+0.1} + \frac{K_3}{s+3-3j} + \frac{\bar{K}_3}{s+3+3j}$

$|K_1| = \frac{(15)(0.12)}{(0.1)(\sqrt{18})(\sqrt{18})} = 1$

$|K_2| = \frac{(15)(0.02)}{(0.1)(4.17)(4.17)} = 0.173$

$|K_3| = \frac{(15)(4.16)}{(\sqrt{18})(4.17)(6)} = 0.588$

(d) Simple pole @ $s = -0.1$: $\quad T = 10$

Complex poles @ $s = -3 \pm 3j$: $\quad T = 1/3$

(e) $e^{-3t}\cos(3t+\theta)$ dominates early on, because it has bigger residue

$e^{-0.1t}$ dominates later on, because it has larger time constant, so it decays slower

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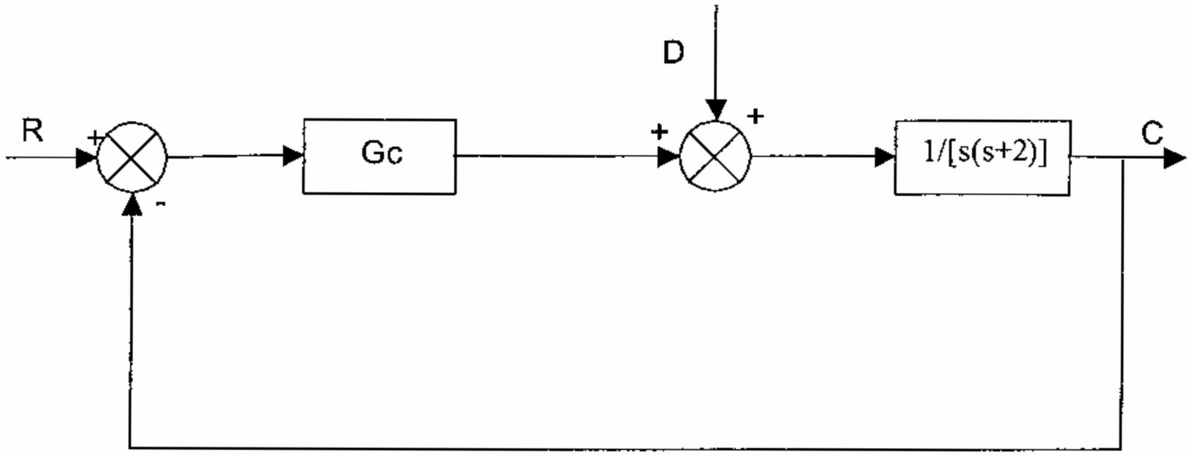
3. Consider the following system with controller G_c, reference input R, and disturbance input D:



a. If G_c is a proportional controller, $G_c = K_c$, what value of K_c will cause the steady-state error for a unit ramp reference input to be 0.1?

b. If we want a zero steady-state error for a unit ramp reference input, what form should the controller G_c take?

c. For a unit step disturbance input, what form of controller G_c should we choose to make the steady-state error zero?

d. Assume that the reference input is a unit step, the disturbance input is zero and we are using a proportional controller. What value (or values) of K_c will yield an underdamped system with a settling time of 4 seconds?

e. Considering the same case as part d, what effect does increasing K_c have on the damping ratio?

(a) $G_c = K_c$ ⇒ type number = 1 (1 integrator in $G(s)$)

⇒ e_{ss} for ramp input $= 1/K$ ← standard gain

Forward path transfer function is: $G(s) = \frac{K_c}{s(s+2)} = \frac{K_c}{2} \cdot \frac{1}{s(\frac{s}{2}+1)}$

⇒ $K = \frac{K_c}{2}$

We need $e_{ss} = \frac{1}{K} = 0.1$ ⇒ $K = 10$ ⇒ $\boxed{K_c = 20}$

(b) To get $e_{ss} = 0$ for unit ramp input, need 2 integrators in $G(s)$

⇒ $G_c(s)$ should have another integrator

(c) To get zero ss error, we need to completely reject the disturbance

$\Rightarrow$ need $C_{ss} = 0$ when D = unit step = $1/s$

$$\frac{C}{D} = \frac{\frac{1}{s(s+2)}}{1+\frac{G_c(s)}{s(s+2)}} \Rightarrow C(s) = \frac{1}{s^2+2s+G_c(s)} \cdot \frac{1}{s}$$

$$\Rightarrow \text{From FVT: } C_{ss} = \lim_{s\to 0} sC(s) = \lim_{s\to 0} \frac{1}{s^2+2s+G_c(s)}$$

$$= \lim_{s\to 0} \frac{1}{G_c(s)}$$

$\Rightarrow$ to get $C_{ss} = 0$, we must have a **$1/s$ term in $G_c(s)$**

(d) Need to find $\frac{C}{R}$:

$$\frac{C}{R} = \frac{\frac{K_c}{s(s+2)}}{1+\frac{K_c}{s(s+2)}} = \frac{K_c}{s^2+2s+K_c} \rightarrow \text{closed-loop poles @ } s = \frac{-2 \pm \sqrt{4(1-K_c)}}{2}$$

$$= -1 \pm \sqrt{1-K_c}$$

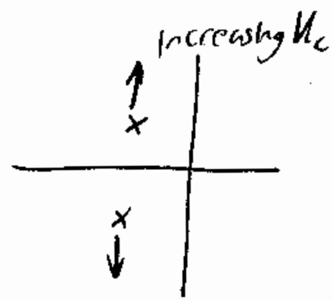
To get an underdamped system, we need complex CL poles

$\Rightarrow$ **$K_c > 1$**

(Note: $K_c > 1 \Rightarrow$ poles at $s = -1 \pm (\,)j \Rightarrow T = 1 \Rightarrow t_s = 4T = 4$ seconds)

(e) Increasing K_c will push the poles away from the real axis

$\Rightarrow$ damping ratio $\downarrow$



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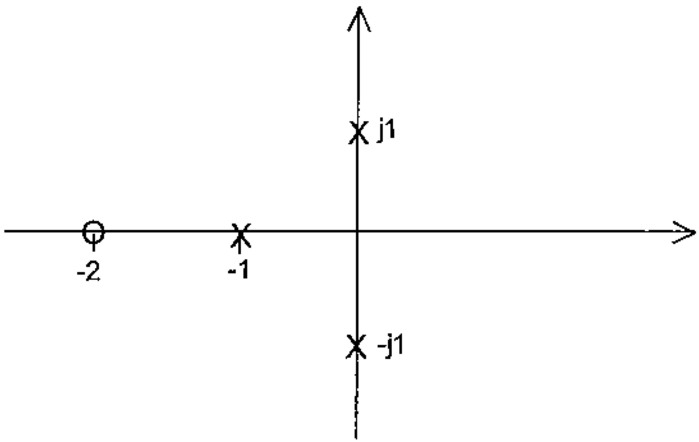
4.

For the system shown below, sketch the root locus for K>0 and K<0 on separate plots. Where appropriate, calculate ρ_0 and the angles of departure of the complex poles.



Leave till later

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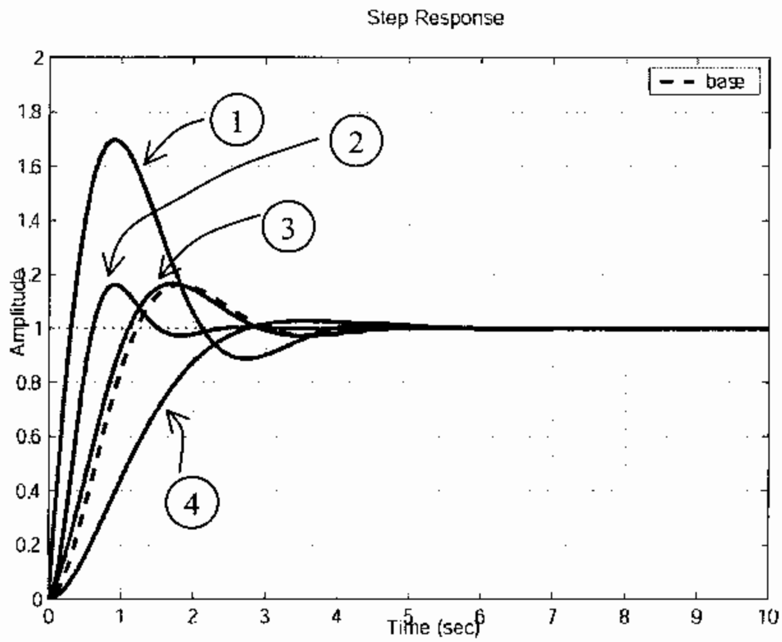
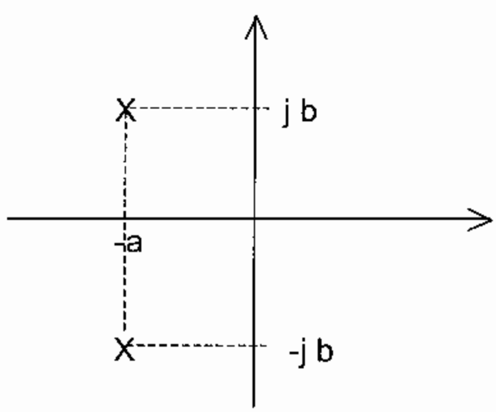
5. The following plot shows the response of 5 different systems to a unit step input. The dashed lines indicate the response of the baseline system.



Consider the baseline pole-zero diagram shown below and the four modified pole-zero diagrams shown on the next page. Each diagram corresponds to one of the responses shown above. For each of the four modified systems, match the step response to the pole-zero diagram. Justify your choice with a few words.

Baseline:



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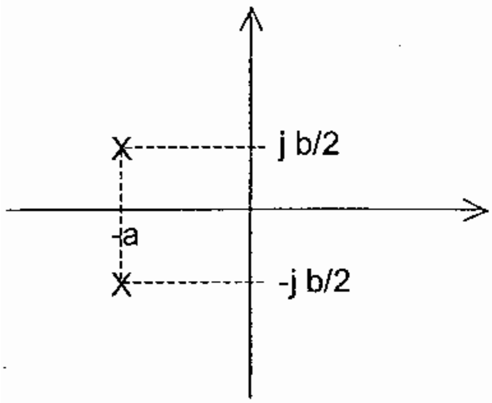
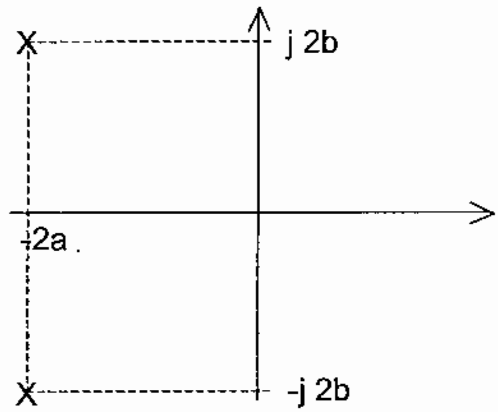
Pole-zero diagram 1:



Which response? ④

Why? ω_d has gone down, ζ has gone up
⇒ lower P.O., longer T_p

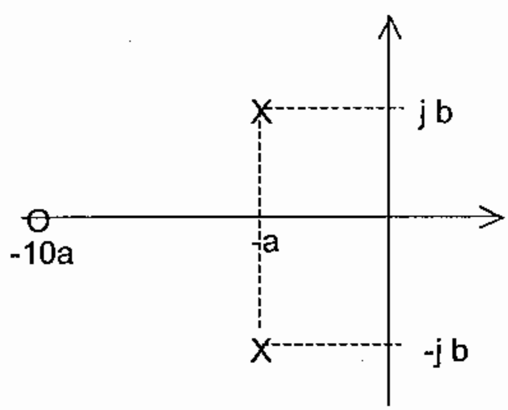
Pole diagram 2:



Which response? ②

Why? ζ same as baseline
⇒ same P.O. as baseline
ω_d has gone up, ⇒ shorter T_p

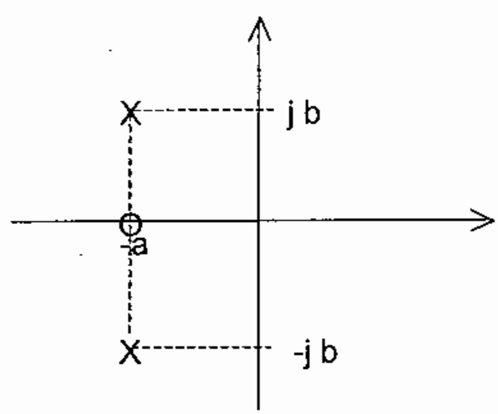
Pole-zero diagram 3:



Which response? 3

Why? Same response as baseline, because zero is too far away to have much effect

Pole diagram 4:



Which response? 1

Why? moving zero to the right causes P.O. ↑, T_p ↓

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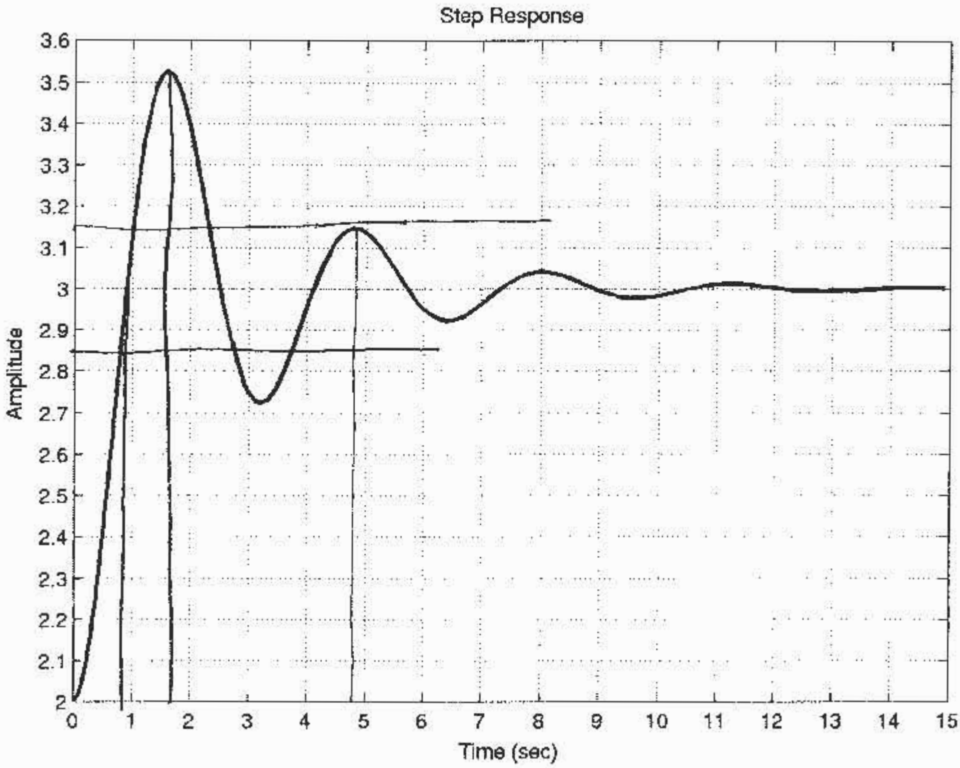
6. The following plot shows the response of a second-order system to a unity step input applied at t=0. At t=0, the system was in its equilibrium position.



a. What is the 5% settling time (T_s)? $\approx$ 3.7 seconds

b. What is the percentage overshoot (P.O.)? $P.O. \approx \frac{0.53}{3} = 18\%$

c. What is the peak time (T_p)? $T_p \approx 1.6$ seconds

d. What is the rise time (T_r)? $t_r \approx 0.9$ seconds

e. What is the damped natural frequency (ω_d)? $\approx$ 3.2 seconds between peaks

$\Rightarrow \omega_d \approx \frac{2\pi}{3.2} \approx 2$ rad/s

f. What are the locations of the poles?

$T \approx \frac{t_s(5\%)}{3} = 1.2$ seconds $= \frac{1}{\zeta\omega_n}$

poles at $-\zeta\omega_n \pm \omega_d j = -0.8 \pm 2j$

Practice State-Space Problem for Quiz 1

16.060: Principles of Automatic Control

October 8, 2003

For the following differential equation:

$$\ddot{w} + 5\dot{w} + 6w = \ddot{r} + \dot{r} + 2r$$

1. Derive a state-space model in the form:

$$\dot{\vec{x}} = A\vec{x} + B\vec{u}$$
$$\vec{y} = C\vec{x} + D\vec{u}$$

   Take r to be the input and w to be the output.

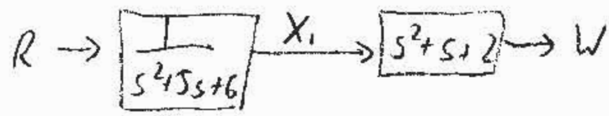
2. Draw a block diagram for the system, and clearly label the input, the state variables, and the output. (Hint: look at the diagram on page 4 of the Lecture 13 notes.)

3. Using your state-space model, determine the transfer function $G(s)$ from the input to the output.

4. Calculate the state-transition matrix $\Phi(t)$.

5. If the initial state is $\vec{x}(0) = \begin{bmatrix}1\\1\end{bmatrix}$, and there are no inputs, what is the state vector $\vec{x}(t)$ at time $t > 0$?

1.) Split into poles and zeros:



$$\Rightarrow \begin{cases} \ddot{x}_1 + 5\dot{x}_1 + 6x_1 = r \\ y = \ddot{x}_1 + \dot{x}_1 + 2x_1 \end{cases}$$

States: x_1, $x_2 = \dot{x}_1$ $\Rightarrow$ $\dot{x}_1 = x_2$, $\dot{x}_2 = -6x_1 - 5x_2 + r$

Input: $u = r$

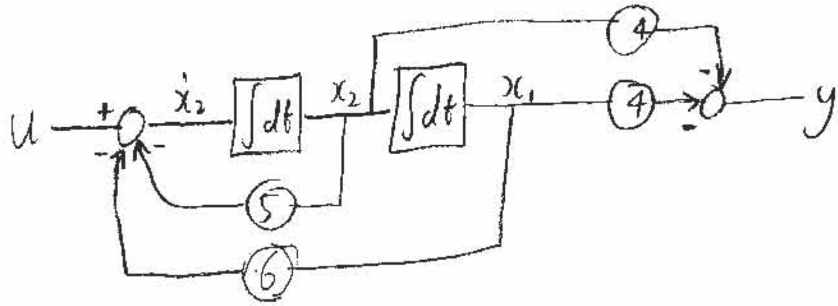
output: $y = w$

$$\Rightarrow \boxed{\dot{\underline{x}} = \begin{bmatrix}0 & 1\\ -6 & -5\end{bmatrix}\underline{x} + \begin{bmatrix}0\\1\end{bmatrix}u}$$

$$y = \ddot{x}_1 + \dot{x}_1 + 2x_1$$
$$= \dot{x}_2 + x_2 + 2x_1$$
$$= (-6x_1 - 5x_2 + r) + x_2 + 2x_1$$
$$= -4x_1 - 4x_2 + r$$

$$\Rightarrow \boxed{y = \begin{bmatrix}-4 & -4\end{bmatrix}\underline{x} + \begin{bmatrix}1\end{bmatrix}u}$$

2.



3. $G(s) = C(sI-A)^{-1}B + D$

$$sI - A = \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix} \Rightarrow (sI-A)^{-1} = \frac{1}{s(s+5)+6}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}$$

$$\Rightarrow G(s) = \begin{bmatrix} -4 & -4 \end{bmatrix} \cdot \frac{1}{s^2+5s+6}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1$$

$$= \begin{bmatrix} -4 & -4 \end{bmatrix} \cdot \frac{1}{(s+3)(s+2)}\begin{bmatrix} 1 \\ s \end{bmatrix} + 1$$

$$= \frac{-4s-4}{(s+3)(s+2)} + 1$$

$$\boxed{G(s) = \frac{s^2+s+2}{s^2+5s+6}}$$

4. $$\Phi(t) = e^{At} = \mathcal{L}^{-1}\left((sI-A)^{-1}\right) = \mathcal{L}^{-1}\left(\begin{bmatrix} \frac{s+5}{(s+3)(s+2)} & \frac{1}{(s+3)(s+2)} \\ \frac{-6}{(s+3)(s+2)} & \frac{s}{(s+3)(s+2)} \end{bmatrix}\right)$$

Use partial fractions:

$$\Phi(t) = \mathcal{L}^{-1}\left(\begin{bmatrix} \frac{-2}{s+3} + \frac{3}{s+2} & \frac{-1}{s+3} + \frac{1}{s+2} \\ \frac{6}{s+3} - \frac{6}{s+2} & \frac{3}{s+3} - \frac{2}{s+2} \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2e^{-3t} + 3e^{-2t} & -e^{-3t} + e^{-2t} \\ 6e^{-3t} - 6e^{-2t} & 3e^{-3t} - 2e^{-2t} \end{bmatrix}$$

5. $$\underset{\sim}{x}(t) = \Phi(t)\, \underline{x}(0) = \begin{bmatrix} \downarrow \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-3t} + 4e^{-2t} \\ 9e^{-3t} - 8e^{-2t} \end{bmatrix}$$