16.06 Lecture 13

More State Space Modeling and Transfer Function Matrices

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Today's Topics

1. More state space modeling.
2. Laplace Transforms for vector/matrix differential equations.
3. An example

A special case for state space models is when there are as many zeros as poles. For example if we have

Recall that we first break $G(s)$ into two parts

The first transfer function relates the input r to the intermediate output x_1.

In the time domain this implies the D.E.

So the state D.E.'s are

Hence

Implying the following block diagrams

However now the output is

which, in the time domain is

If we go back to the block diagram we see that we must feed the $\dot{x}_2$ signal forword to get the output

We can then write the equation for w

Laplace Transforms of State Vectors

With appropraite definitions of terms we can usefully apply Laplace Transform methods to vector/matrix differential equations. Thus we define the Laplace Transform of a state vector as

It then follows naturally that the L.T. of the derivative of $\underline{x}(t)$ is

Thus if we have the state equation in the time domain

we can take the L.T. of both sides to obtain

which can also be written as

where I is the identity matrix

We can then solve for the L.T. of the state as

Now we also have the output equation

which can also be transformed to obtain the L.T. of the output vector

Upon substitution of the equation for the L.T. of the state obtains

If we are only interested in the response to inputs we can obtain a matrix transfer function

where

Example

Earlier we turned the scalar transfer function

into a state space model and found the following matrices

Then

Its inverse is

Note that the denominater is the left hand side of the characteristic equation

Also