MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics
16.060: Principles of Automatic Control
Fall 2003

PROBLEM SET 10

This problem set is for practice only!

Problem 1: Sketching Bode Plots

Sketch (by hand) the Bode plot for each of the following systems. *Hint*: first rewrite each transfer function in Bode (standard) form.

1. $G(s) = \frac{1}{s+1}$
2. $G(s) = \frac{1}{s(s+1)}$
3. $G(s) = \frac{10}{s^2+s+10}$
4. $G(s) = \frac{s+10}{s+1}$
5. $G(s) = \frac{s+1}{s+10}$
6. $G(s) = \frac{1000(0.1s+1)}{s(s+1)(s^2+10s+100)}$

Problem 2: Bode Plots and Polar Plots

As you probably have noticed already, Bode plots and polar plots represent the same information. (Recall that the polar plot of a transfer function is just the Nyquist plot for $0 < \omega < \infty$.) Nyquist plots are essential for evaluating the stability of the closed-loop system when the open-loop transfer function has poles in the right half plane. Bode plots are useful for designing controllers because they can be sketched quickly and accurately by hand, without difficult calculations.

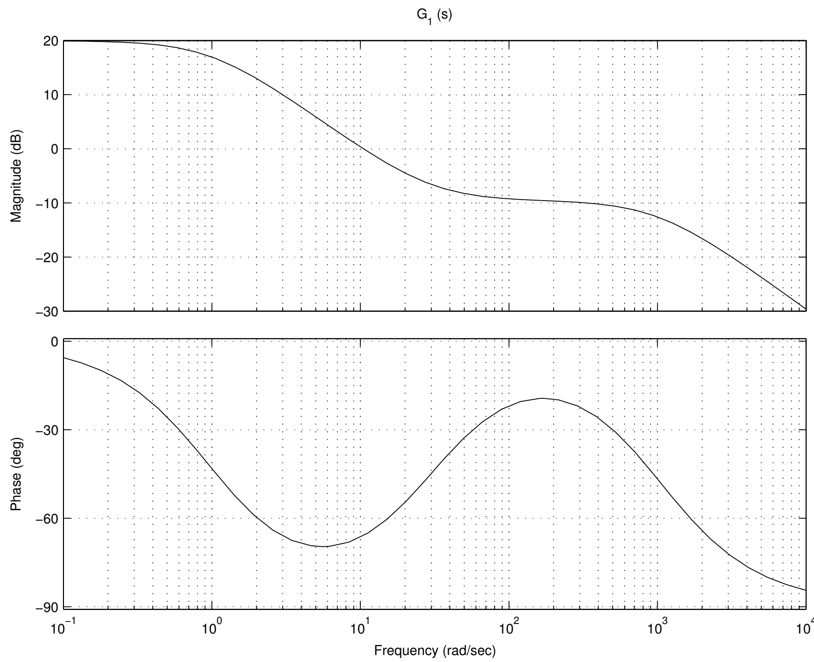
Consider

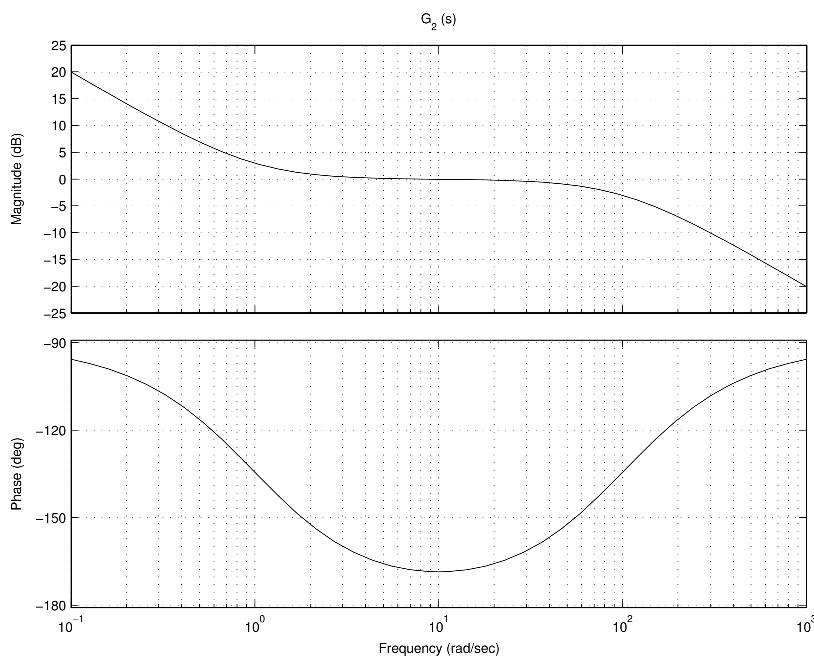
$$G(s) = \frac{100}{s(s+4)(s^2+20s+100)}$$

1. Sketch the Bode plot
2. Sketch the polar plot
3. Label the gain and phase margins on each of these. (Do not compute the actual values).

Problem 3: System Identification from Bode Plots

Two bode plots are shown below. Identify the transfer function corresponding to each plot.



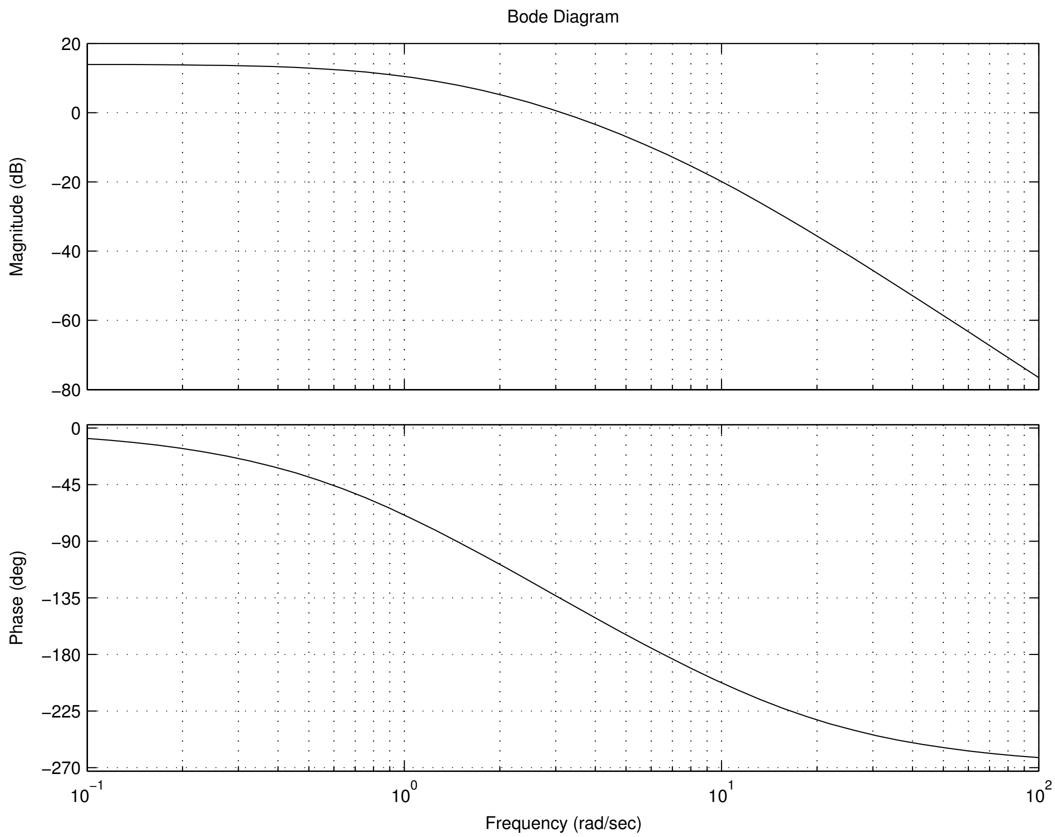


Problem 4: Designing a Proportional Controller using Bode

For many open-loop transfer functions $G(s)$, the closed-loop transfer function $\frac{G}{1+G}$ can be modeled approximately as a second-order transfer function $\frac{\omega_n^2}{s^2+2\zeta\omega_n+\omega_n^2}$. The damping ratio ζ and the natural frequency ω_n of the closed-loop system can be determined from the phase margin ϕ_m and the crossover frequency ω_c of the open-loop system. Figure 8.7 in Van de Vegte shows the relation between ζ and ϕ_m, and Figure 8.6 shows the relation between ω_c and ω_n as a function of ζ. Table 8.4.1 may also be useful.

The Bode plot for the following open-loop transfer function is shown below, and the data for this Bode plot are tabulated on the next page.

$$G(s) = \frac{150}{(s+1)(s+3)(s+10)}$$



1. Using the graph and/or the table of values, find the crossover frequency ω_c, the gain margin GM, and the phase margin ϕ_m of $G(s)$.
2. If you closed the loop around $G(s)$, what would be the damping ratio and natural frequency of the closed-loop system?
3. Suppose we wanted to increase the damping ratio of the closed-loop system to 0.6. What should the phase margin of the open-loop system be?
4. Adding a proportional controller K does not change the Bode phase plot, but it will shift the Bode magnitude plot up or down depending on the value of K. That has the effect of shifting the crossover frequency, and therefore changing the phase margin as well. What should be the gain of the proportional controller K so that the phase margin of $KG(s)$ comes out to the value you calculated in the previous part?

5. With this proportional controller, what is the new natural frequency of the closed-loop system?

Bode data for $G(s)$ in Problem 4

ω	$\lvert G\rvert$	$\angle G$
0.1000	4.9722	-8.1927
0.1184	4.9611	-9.6868
0.1401	4.9458	-11.4491
0.1658	4.9245	-13.5250
0.1962	4.8951	-15.9657
0.2322	4.8546	-18.8281
0.2748	4.7994	-22.1738
0.3252	4.7246	-26.0664
0.3849	4.6249	-30.5682
0.4556	4.4939	-35.7344
0.5391	4.3254	-41.6055
0.6381	4.1144	-48.1999
0.7552	3.8584	-55.5072
0.8938	3.5587	-63.4861
1.0578	3.2215	-72.0682
1.2519	2.8576	-81.1680
1.4816	2.4809	-90.6937
1.7535	2.1063	-100.5559
2.0753	1.7478	-110.6698
2.4561	1.4168	-120.9525
2.9068	1.1217	-131.3198
3.4402	0.8674	-141.6865
4.0715	0.6552	-151.9708
4.8187	0.4837	-162.0984
5.7029	0.3492	-172.0037
6.7494	0.2467	-181.6253
7.9880	0.1706	-190.8980
9.4539	0.1156	-199.7480
11.1887	0.0768	-208.0942
13.2419	0.0501	-215.8570
15.6719	0.0322	-222.9709
18.5478	0.0204	-229.3950
21.9515	0.0128	-235.1179
25.9797	0.0079	-240.1561
30.7472	0.0049	-244.5482
36.3895	0.0030	-248.3471
43.0672	0.0018	-251.6130
50.9703	0.0011	-254.4076
60.3237	0.0007	-256.7907
71.3935	0.0004	-258.8178
84.4947	0.0002	-260.5389
100.0000	0.0001	-261.9981