16.06 Lecture 24
Compensator Design

Karen Willcox

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Today's Topics

1. Phase-lead compensator design

2. Phase-lag compensator design

Reading: 6.6, 6.7

1 Phase-lead compensator design

We have the following system:

where

Consider the pole-zero diagram:

Phase-lead compensation has a stabilizing effect.

At A_1, angle of plant =

At A_2, angle of plant + angle of compensator =

At A_2, angle of plant will be

So the compensator must contribute

From the triangle, we have

Now we can establish a design procedure.

2 Design Procedure

7 steps on handout.

insert phase-lead design procedure here

3 Phase-lead compensator design example

$$G(s) = \frac{1}{s(s+2)}$$

Design $G_c(s)$ to meet the following closed-loop specifications:

(i) Dominant time constant should be 0.25 sec

(ii) P.O. for step response $\approx 16\%$

- Step 1

  Specifications imply:

  Therefore, the dominant closed-loop poles are at

- Step 2 Angle condition:

- Step 3

  Place zero at -4

- Step 4

$$\tan 46.1^\circ =$$

- Step 5

  Magnitude condition:

$$K_c =$$

- Step 6

$$G_cG =$$

$$\text{loop gain, } K =$$

$$e_{ss} =$$

$$P.O. =$$

  Third pole at

- Step 7

  P.O. is too high.

  Move zero and pole to the left and try again.

4 Phase-lag compensator design

The transfer function for phase-lag compensation is the same as for phase-lead, but now $z > p$, so the pole is closer to the origin that the zero.

While phase-lead is stabilizing, phase-lag is often used to

The compensator transfer function is:

$$G_c(s) =$$

Draw the pole-zero diagram:

The pole-zero pair is added close to the origin, far from the desired closed-loop poles. The vectors $(s+z)$ and $(s+p)$ from the pair to the desired poles almost cancel each other, so that the net contribution to the vector angle is small. Therefore, the loci change just a little in the region of interest.

Why use this compensation if the loci don't change much? Consider the compensator transfer function:

$$G_c(s) =$$

- The root locus gain is
- The gain used in the steady-state error calculation is
- By placing the pole much closer to the origin than the zero, we can
- But then we will have a closed-loop pole close to the origin (a slow pole)! This is okay because

The design procedure for a phase-lag compensator is as follows:

1. Draw the root loci for a proportional gain controller
2. Determine the desired position of the dominating pair of closed-loop poles on these loci from the specifications.
3. Determine the root locus gain at the position (using the magnitude condition) and hence the value of K_c for P-control.
4. For this value of K_c, determine the value of the factor z/p needed to satisfy the specifications on steady-state accuracy.
5. Choose p and z with this ratio and close enough to the origin that the vector angles to the dominant poles differ only a few degrees.
6. Draw the loci of the compensated system and find the dominating poles. Reduce K_c if needed to counter any reduction of the relative stability.

Phase-lag compensator design example:

$$G(s) = \frac{1}{s(s+2)}$$

Design $G_c(s)$ to meet the following closed-loop specifications:

(i) $\zeta = 0.5$

(ii) Steady-state error less than 5%

- Step 1

- Step 2

  From specifications: closed-loop poles are at

- Step 3

  Magnitude condition:

  $$K_c =$$

- Step 4

  Loop gain function is:

  Loop gain is:

  $e_{ss} =$

  To satisfy error specification, must increase loop gain to

  Therefore, $z/p =$

- Step 5

  Choose

  $$G_c =$$

  Draw the new root locus: