16.06 Lecture 6
The s-Plane, Poles and Zeroes

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September 15, 2003

Today's Topics

1. Poles and zeroes

2. Transient response and inverse Laplace transform

3. Graphical determination of residues

Reading: 1.7 (from the top of pg. 14), 1.8, 1.9

1 The s-plane

We write

$$C(s) = G(s)R(s)$$

where C, G and R are each ratios of polynomials in s, i.e. $G(s) = \frac{\text{num } G}{\text{den } G}$.

Consider the following definitions:

- **zeroes** of C, G and R are
- **poles** of C, G and R are
- **system zeroes and poles** are
- **system characteristic polynomial** is
- **system characteristic equation** is

Note that the roots of the C.E. are

Since, the polynomials have real coefficients, the poles and zeros are

We plot the poles and zeros in the $s(\sigma + j\omega)$ plane.

Example:

Assume $R(s) = \frac{1}{s}$. Then the pole-zero pattern of $C(s) = R(s)G(s)$ is the superposition of the patterns of $R(s)$ and $G(s)$:

$$C(s) = \frac{K(s+2)}{s(s+4)}$$

2 Transient response and inverse Laplace transform

Question: Given the pole-zero diagram of $C(s)$, how do you get $c(t)$?

Answer: perform inverse Laplace transform

(a) If $C(s)$ is simple, use a table.

If $C(s)$ is complicated, we can use partial fraction expansion (PFE).

Example:

(b) There is an alternative approach, which will serve us well in the future

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3 Graphical determination of residues (real poles)

(a) Typical factor in PFE is

(a is positive and b is negative)

We can write

where $b - (-a)$ is

So in the s-plane:

(b) The general expression for K_1 in the example above is

(c) Using the actual values, we have:

and for K_2:

(d) So as before,

$$c(t) = K(\frac{1}{2} + \frac{1}{2}e^{-4t})$$

4 Root locus gain (vdv pg. 20)

Definition: The *root locus gain* of a transform or a transfer function is that which results if the coefficients of the highest powers of s in the numerator and denominator polynomials are made equal to unity.

For, say,

$$C(s) = 2\frac{0.5s + 1}{(s + 3)(0.1s + 1)} = \frac{2(0.5)}{0.1}\frac{s + 2}{(s + 3)(s + 10)}$$

the root locus gain is 2(0.5/0.1)=10.

Reviewing the expressions for K_1, K_2 and K_3 reveals the following general rule.

Graphical Residue Rule: The residue K_i at the pole $-p_i$ of $C(s)$ equals the root locus gain times the product of the vectors from all zeros of $C(s)$ to $-p_i$ divided by the product of the vectors from all other poles of $C(s)$ to $-p_i$.