

16.06 Lecture 35

Bode Diagrams

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Today's Topics

1. Introduction to Bode diagrams
2. Units
3. Gain Factor
4. Integrator Factors
5. Simple Lag

Reading: 7.5

1 Bode Diagrams

Bode diagrams are an alternative to polar plots and are very widely used. You will see that these plots are easier to make than polar plots. We also see that we can quickly determine different aspects of system performance from a Bode diagram.

Consider the general frequency response function $G(j\omega)$. We can write

$$G(j\omega) = \tag{1}$$

or

$$\ln G(j\omega) = \tag{2}$$

Recall that in a polar plot, we plotted magnitude versus phase with frequency as a parameter. Bode suggested plotting two separate curves:

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These two curves taken together are called the Bode diagram.

2 Units

- The tradition is not to use logs to the base e but rather logs to the base 10. They relate to each other through a multiplicative factor:

$$\ln x = 2.3026 \log x$$

- For the magnitude, we will work in units of decibels:

$$M \text{ in dB} =$$

Note that if $M = M_1 M_2 M_3$, then

$$\log M =$$

and

$$M_{dB} =$$

- If $M = 1$, then $M_{dB} =$
- If $M = 10$, then $M_{dB} =$
- If $M = 0.1$, then $M_{dB} =$
- The frequency is plotted on a

3 Bode Diagram Construction

Remember that all of our systems consist of integrators plus poles and zeroes that are either real or complex conjugate pairs (i.e. a combination of simple and quadratic lags). We can therefore write the frequency response function as

$$G(j\omega) = \tag{3}$$

Consider each of the four kinds of factors in this equation:

1. K is
2. $1/(j\omega)^n$ represents
3. S_i and $1/S_i$ are respectively

$$S_i =$$

4. Q_i and $1/Q_i$ are respectively

$$Q_i =$$

If we consider taking the log of the frequency response function, then we can sum the contributions from each of these four factors:

$$\log |G(j\omega)| = \tag{4}$$

$$\phi = \tag{5}$$

4 Gain Factor

Consider $K > 0$:

$$M_{dB} =$$

$$\phi =$$

5 Integrator Factors

Consider $\frac{1}{(j\omega)^n}$:

$$M_{dB} =$$

$$=$$

- At $\omega = 1$,
- At $\omega = 10$,
- At $\omega = 100$,
- On a log scale, the magnitude plot is

- Phase angle:
- If we have differentiator factors $(j\omega)^n$, then the plots are just the mirror images of the integrator plots relative to the 0dB and 0° axes.

Bode diagram for integrator factors:

6 Simple Lag

Consider a simple lag term: $\frac{1}{S} = \frac{1}{j\omega T + 1}$

$$M =$$

$$M_{dB} =$$

$$\phi =$$

- If $\omega \ll 1/T$, ωT

$$M$$

$$M_{dB}$$

$$\phi \rightarrow$$

This gives us the

- If $\omega \gg 1/T$, ωT

$$M$$

$$M_{dB}$$

$$\phi \rightarrow$$

This gives us the

Draw the asymptotic approximation:

The asymptotes meet at the **break frequency** or **corner frequency**.

- If $\omega = 1/T$

$$\omega T =$$

$$M =$$

$$M_{dB} =$$

$$\phi =$$

Insert Bode diagram for a simple lag

- Some details of first-order factors:
 1. The true curves depart from the asymptotic approximations by $\pm 0.15 \log$ units or ± 3 dB at $\omega = 1/T$.
 2. One octave below the break frequency the angle is -26.6°
 3. One octave above the break frequency the angle is $-90 + 26.6 = -63.4^\circ$
 4. One decade below the break frequency the angle is -5.7°
 5. One decade above the break frequency the angle is $-90 + 5.7 = 84.3^\circ$