16.06 Lecture 27

Polar Plots

November 6, 2003

Today's Topics:

1. First order system polar plot
2. Second order system polar plots
3. Other examples

Recall from last time that if the system $G(s)$ receives a steady sinusoidal input of amplitude A, so

then the steady state output will be a sinusoid of magnitude $A \cdot M(\omega)$ with its phase shifted by $\phi(\omega)$

where

Now let's pause for a bit and think about some interpretations of $G(j\omega)$.

$G(j\omega)$=the transfer function $G(s)$ evaluated at the point $s \quad j\omega$ on the imaginary axis of the "s" plane.

In particular, if

then

Now the term $(j\omega - z_1)$ is the vector from the zero at z_1 to the point $j\omega$ on the imaginary axis of the "s" plane

Similarly, if the poles p_1 and p_2 are complex conjugates then

In general

where-

k_{rl} root locus gain

A_{z_i}=magnitude of the vector from the zero at $-z_i$ to the point $j\omega$

A_{p_i} magnitude of the vector from the pole at $-p_i$ to the point $j\omega$

ϕ_{z_i} angle of the vector from the zero at $-z_i$ to the point $j\omega$

ϕ_{p_i} angle of the vector from the pole at $-p_i$ to the point $j\omega$

Example-first order system

As ω goes from zero to plus infinity $G(j\omega)$ traces a contour in the complex plane

Example: second order system

Some typical second order system polar plots for

$$G(s) = \frac{k_{rl}}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Critical damping ($\delta = 1$)-

Medium damping ($\delta = 0.7$)

Light damping(δ 0.1)

Zero damping (δ 0)

The Quanser

An integrator (pole at the origin)

A differentiator (zero at the origin)