16.06 Lecture 36
Bode Diagrams

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December 3, 2003

Today's Topics

1. Simple Lag

2. Quadratic Lag

3. Bode diagram construction

4. Bode diagram examples

Reading: 7.5

1 Simple Lag

Consider a simple lag term: $\frac{1}{S} = \frac{1}{j\omega T + 1}$

$$M =$$

$$M_{dB} =$$

$$\phi =$$

- If $\omega << 1/T$, $\omega T << 1$

$$M$$

$$M_{dB}$$

$$\phi \rightarrow$$

This gives us the

- If $\omega >> 1/T$, $\omega T >> 1$

$$M$$

$$M_{dB}$$

$$\phi \rightarrow$$

This gives us the

Draw the asymptotic approximation:

The asymptotes meet at the **break frequency** or **corner frequency**.

- If $\omega = 1/T$

$$\omega T =$$

$$M =$$

$$M_{dB} =$$

$$\phi =$$

Insert Bode diagram for a simple lag

- Some details of first-order factors:

    1. The true curves depart from the asymptotic approximations by ± 0.15 log units or ± 3 dB at $\omega = 1/T$.

    2. One octave below the break frequency the angle is $-26.6°$

    3. One octave above the break frequency the angle is $-90 + 26.6 = -63.4°$

    4. One decade below the break frequency the angle is $-5.7°$

    5. One decade above the break frequency the angle is $-90 + 5.7 = 84.3°$

2 Quadratic Lag

Consider a quadratic lag term:

$$\frac{1}{Q} = \frac{1}{(j\omega/\omega_n)^2 + 2\zeta j\omega/\omega_n + 1}$$

M =

M_{dB} =

ϕ =

- If $\omega/\omega_n << 1$

  M

  M_{dB}

  ϕ

  This gives us the

- If $\omega/\omega_n >> 1$

  M

  M_{dB}

  ϕ

  This gives us the

Draw the asymptotic approximation:

The asymptotes meet at the break frequency or corner frequency.

- If $\omega/\omega_n = 1$

$$M = \frac{1}{2\zeta}$$

$$M_{dB} = 20log\frac{1}{2\zeta}$$

$$\phi = -90^\circ$$

  Smaller damping ratios cause a larger peak in M_{dB} and a more abrupt change in ϕ.

- Note that for a quadratic lag, the actual magnitude plot can be very different to the asymptotic approximation.

Insert the Bode diagram for a quadratic lag

3 Examples

General approach:

1. Determine which elementary factors are present and calculate any break frequencies.

2. Plot the low-frequency asymptote.

3. Add in the other elementary factors in order of increasing break frequencies. Note that each factor has a 0dB asymptote below its break frequency, and so does not affect the asymptotic plot below this frequency.

    - A simple lag causes a change in slope of -20 dB/dec
    - A quadratic lag causes a change in slope of -40 dB/dec

Example 1:

$$G(s) = \frac{10(0.1s + 1)}{s(0.01s + 1)}$$

Example 2:

$$G(s) = \frac{(-0.1s + 1)}{0.1s + 1}$$

Example 3:

$$G(s) = \frac{10(s+1)}{s(10s+1)}$$